

Sensor Fusion - Test Bench

Dehaeze Thomas

February 2, 2021

Contents

1	Experimental Setup	4
2	First identification of the system	7
2.1	Load Data	7
2.2	Excitation Signal	7
2.3	Identified Plant	7
2.4	Simscape Model - Comparison	8
2.5	Integral Force Feedback	8
2.6	Inertial Sensors	13
3	Optimal IFF Development	18
3.1	Load Data	18
3.2	Experimental Data	18
3.3	Model of the IFF Plant	20
3.4	Root Locus and optimal Controller	20
3.5	Verification of the achievable damping	21
4	Generate the excitation signal	24
4.1	Transfer function from excitation signal to displacement	24
4.2	Motion measured during Huddle test	25
5	Identification of the Inertial Sensors Dynamics	29
5.1	Load Data	29
5.2	Compare PSD during Huddle and and during identification	29
5.3	Compute transfer functions	31
6	Inertial Sensor Noise and the \mathcal{H}_2 Synthesis of complementary filters	34
6.1	Load Data	34
6.2	ASD of the Measured displacement	34
6.3	ASD of the Sensor Noise	35
6.4	Noise Model	36
6.5	\mathcal{H}_2 Synthesis of the Complementary Filters	37
6.6	Results	38
7	Inertial Sensor Dynamics Uncertainty and the \mathcal{H}_∞ Synthesis of complementary filters	40
7.1	Load Data	40
7.2	Compute the dynamics of both sensors	40
7.3	Dynamics uncertainty estimation	41
7.4	\mathcal{H}_∞ Synthesis of Complementary Filters	41
7.5	Obtained Super Sensor Dynamical Uncertainty	46
8	Optimal and Robust sensor fusion using the $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis	47
8.1	Noise and Dynamical uncertainty weights	47
8.2	Obtained Super Sensor Noise	47
8.3	Obtained Super Sensor Dynamical Uncertainty	48

8.4	Experimental Super Sensor Dynamical Uncertainty	48
8.5	Experimental Super Sensor Noise	48
9	Matlab Functions	51
9.1	<code>createWeight</code>	51
9.2	<code>plotMagUncertainty</code>	51
9.3	<code>plotPhaseUncertainty</code>	52

In this document, we wish the experimentally validate sensor fusion of inertial sensors.

This document is divided into the following sections:

- Section 1: the experimental setup is described
- Section 2: a first identification of the system dynamics is performed
- Section 3: the integral force feedback active damping technique is applied on the system
- Section 4: the optimal excitation signal is determine in order to have the best possible system dynamics estimation
- Section 5: the inertial sensor dynamics are experimentally estimated
- Section 6: the inertial sensor noises are estimated and the \mathcal{H}_2 synthesis of complementary filters is performed in order to yield a super sensor with minimal noise
- Section 7: the dynamical uncertainty of the inertial sensors is estimated. Then the \mathcal{H}_∞ synthesis of complementary filters is performed in order to minimize the super sensor dynamical uncertainty
- Section 8: Optimal sensor fusion is performed using the $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis

1 Experimental Setup

The goal of this experimental setup is to experimentally merge inertial sensors. To merge the sensors, optimal and robust complementary filters are designed.

A schematic of the test-bench used is shown in Figure 1.1 and a picture of it is shown in Figure 1.2.

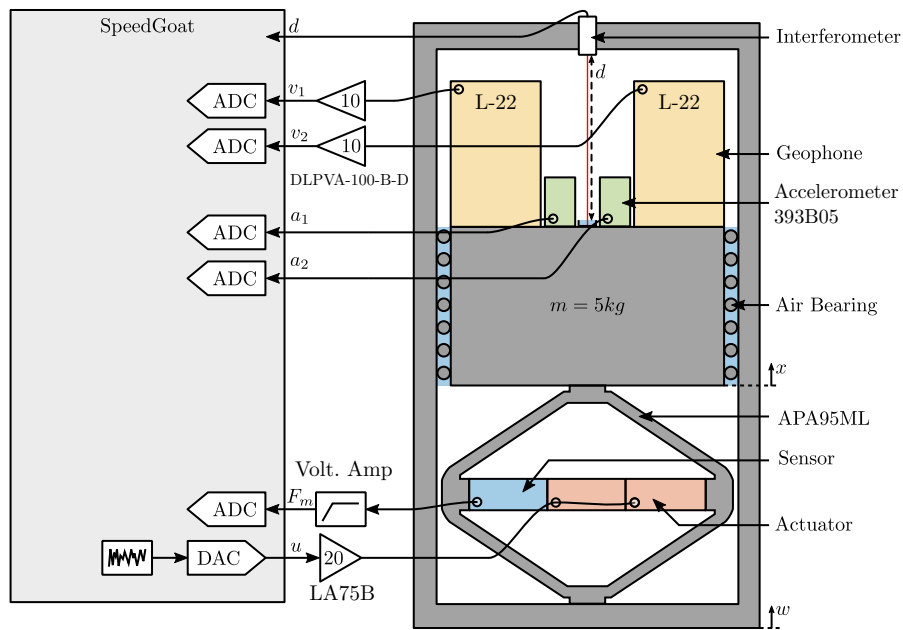


Figure 1.1: Schematic of the test-bench

Two inertial sensors are used:

- An vertical accelerometer *PCB 393B05* ([doc](#))
- A vertical geophone *Mark Product L-22*

Basic characteristics of both sensors are shown in Tables 1.1 and 1.2.

Table 1.1: Accelerometer (393B05) Specifications

Specification	Value
Sensitivity	1.02 [V/(m/s ²)]
Resonant Frequency	> 2.5 [kHz]
Resolution (1 to 10kHz)	0.00004 [m/s ² rms]

The ADC used are the IO131 Speedgoat module ([link](#)) with a 16bit resolution over +/- 10V.

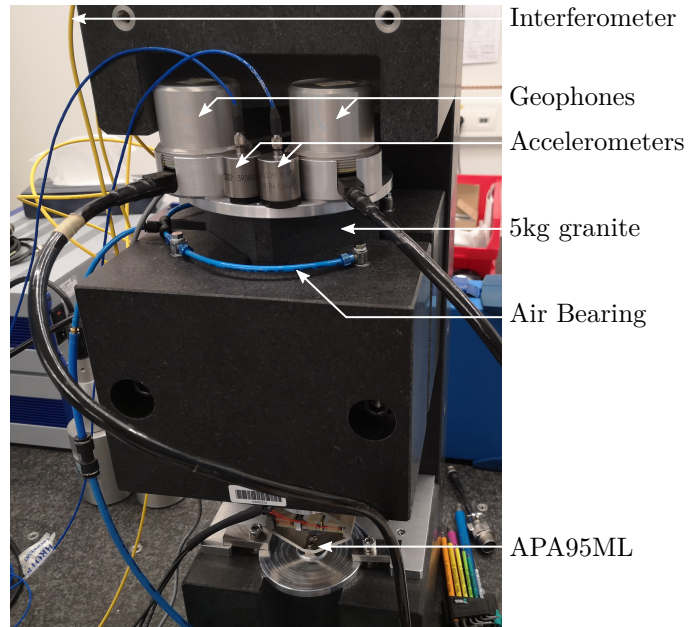


Figure 1.2: Picture of the test-bench

Table 1.2: Geophone (L22) Specifications

Specification	Value
Sensitivity	To be measured [V/(m/s)]
Resonant Frequency	2 [Hz]

The geophone signals are amplified using a DLPVA-100-B-D voltage amplifier from Femto ([doc](#)). The force sensor signal is amplified using a Low Noise Voltage Preamplifier from Ametek ([doc](#)).

The excitation signal is amplified by a linear amplifier from Cedrat (LA75B) with a gain equal to 20 ([doc](#)).

Geophone electronics:

- gain: 10 (20dB)
- low pass filter: 1.5Hz
- high pass filter: 100kHz (2nd order)

Force Sensor electronics:

- gain: 10 (20dB)
- low pass filter: 1st order at 3Hz
- high pass filter: 1st order at 30kHz

2 First identification of the system

In this section, a first identification of each elements of the system is performed. This include the dynamics from the actuator to the force sensor, interferometer and inertial sensors.

Each of the dynamics is compared with the dynamics identified form a Simscape model.

2.1 Load Data

The data is loaded in the Matlab workspace.

```
id_ol = load('identification_noise_bis.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

Then, any offset is removed.

```
id_ol.d = detrend(id_ol.d, 0);  
id_ol.acc_1 = detrend(id_ol.acc_1, 0);  
id_ol.acc_2 = detrend(id_ol.acc_2, 0);  
id_ol.geo_1 = detrend(id_ol.geo_1, 0);  
id_ol.geo_2 = detrend(id_ol.geo_2, 0);  
id_ol.f_meas = detrend(id_ol.f_meas, 0);  
id_ol.u = detrend(id_ol.u, 0);
```

2.2 Excitation Signal

The generated voltage used to excite the system is a white noise and can be seen in Figure 2.1.

2.3 Identified Plant

The transfer function from the excitation voltage to the mass displacement and to the force sensor stack voltage are identified using the `tfestimate` command.

```
Ts = id_ol.t(2) - id_ol.t(1);  
win = hann(ceil(10/Ts));
```

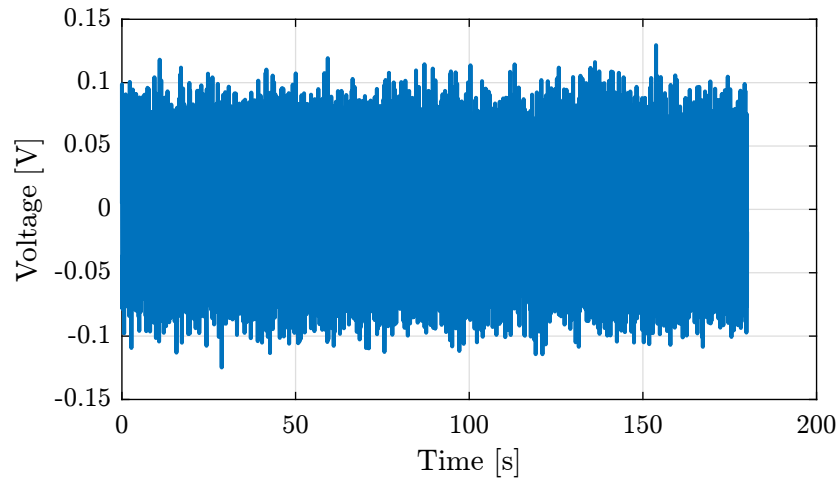


Figure 2.1: Voltage excitation signal

```

Matlab
[tf_fmeas_est, f] = tfestimate(id_ol.u, id_ol.f_meas, win, [], [], 1/Ts); % [V/V]
[tf_gol_est, ~] = tfestimate(id_ol.u, id_ol.d, win, [], [], 1/Ts); % [m/V]

```

The bode plots of the obtained dynamics are shown in Figures 2.2 and 2.3.

2.4 Simscape Model - Comparison

A Simscape model representing the test-bench has been developed. The same transfer functions as the one identified using the test-bench can be obtained thanks to the Simscape model.

They are compared in Figure 2.4 and 2.5. It is shown that there is a good agreement between the model and the experiment.

```

Matlab
load('piezo_amplified_3d.mat', 'int_xyz', 'int_i', 'n_xyz', 'n_i', 'nodes', 'M', 'K');

```

2.5 Integral Force Feedback

The force sensor stack can be used to damp the system. This makes the system easier to excite properly without too much amplification near resonances.

This is done thanks to the integral force feedback control architecture.

The force sensor stack signal is integrated (or rather low pass filtered) and fed back to the force sensor stacks.

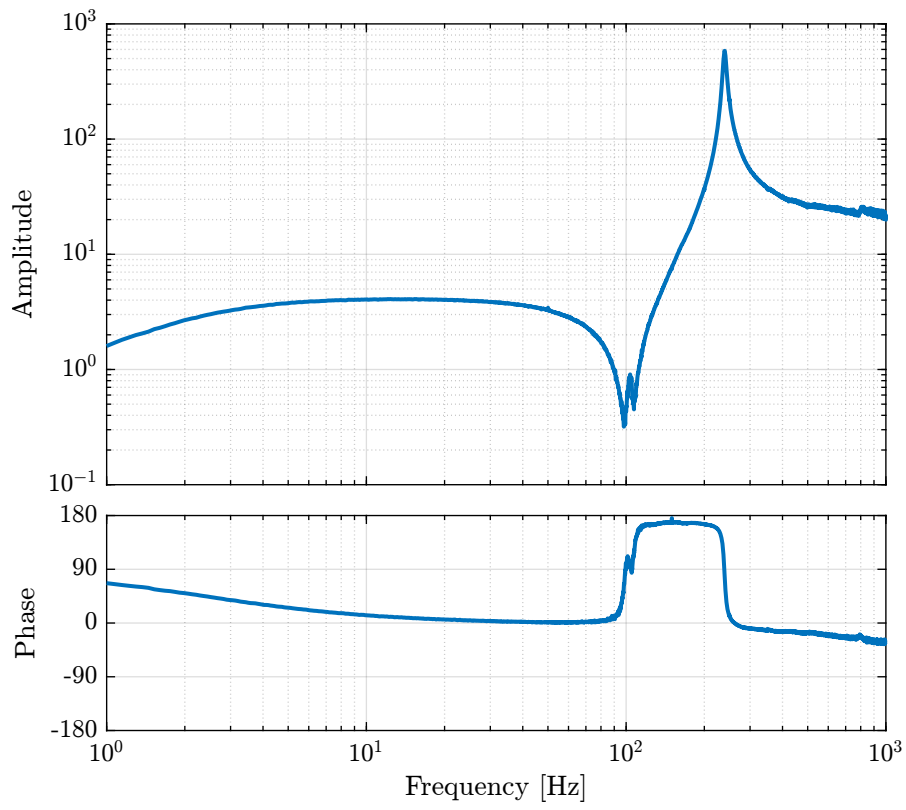


Figure 2.2: Bode plot of the dynamics from excitation voltage to measured force sensor stack voltage

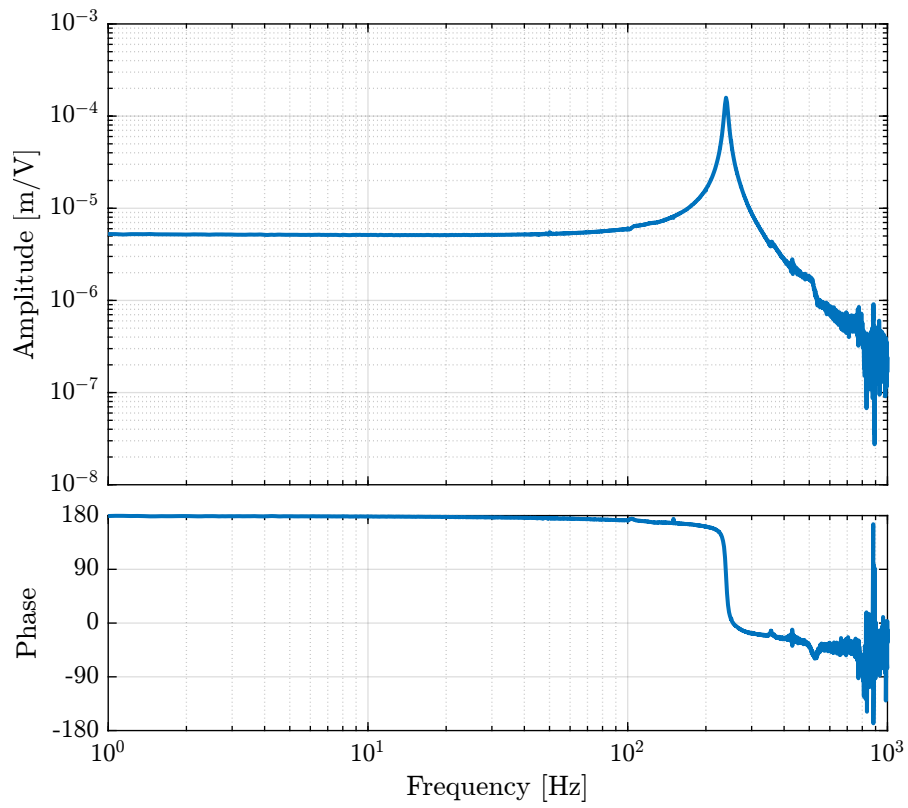


Figure 2.3: Bode plot of the dynamics from excitation voltage to displacement of the mass as measured by the interferometer

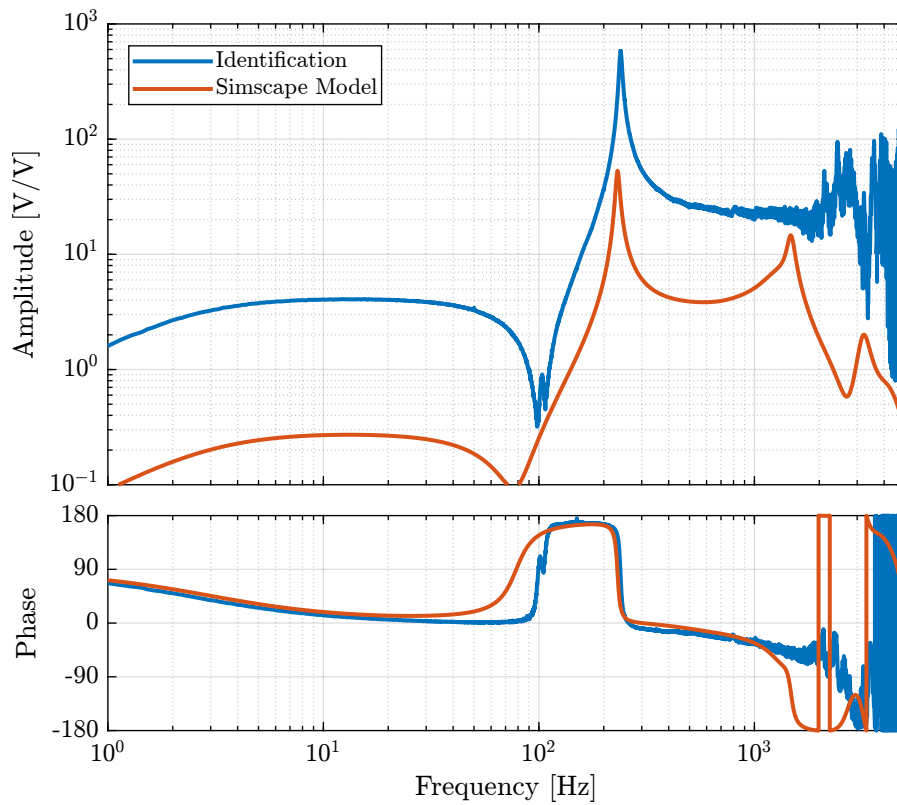


Figure 2.4: Comparison of the dynamics from excitation voltage to measured force sensor stack voltage - Identified dynamics and Simscape Model

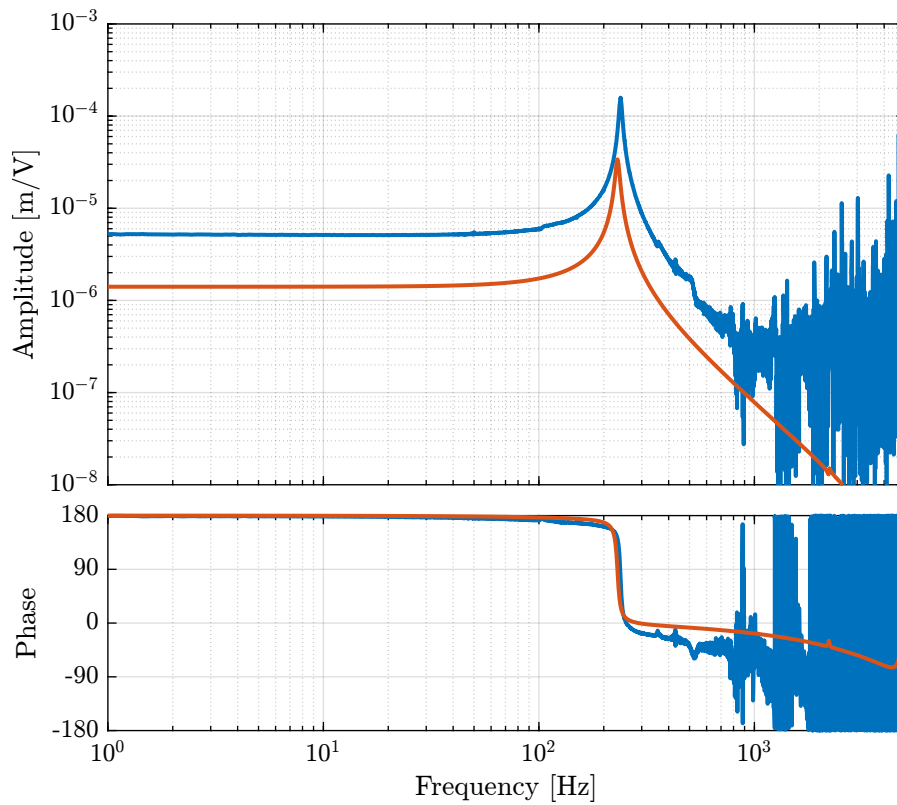


Figure 2.5: Comparison of the dynamics from excitation voltage to measured mass displacement - Identified dynamics and Simscape Model

The low pass filter used as the controller is defined below:

```
Matlab
Kiff = 102/(s + 2*pi*2);
```

The integral force feedback control strategy is applied to the Simscape model as well as to the real test bench.

The damped system is then identified again using a noise excitation.

The data is loaded into Matlab and any offset is removed.

```
Matlab
id_cl = load('identification_noise_iff_bis.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');

id_cl.d = detrend(id_cl.d, 0);
id_cl.acc_1 = detrend(id_cl.acc_1, 0);
id_cl.acc_2 = detrend(id_cl.acc_2, 0);
id_cl.geo_1 = detrend(id_cl.geo_1, 0);
id_cl.geo_2 = detrend(id_cl.geo_2, 0);
id_cl.f_meas = detrend(id_cl.f_meas, 0);
id_cl.u = detrend(id_cl.u, 0);
```

The transfer functions are estimated using `tfestimate`.

```
Matlab
[tf_G_cl_est, ~] = tfestimate(id_cl.u, id_cl.d, win, [], [], 1/Ts);
[co_G_cl_est, ~] = mscohere(id_cl.u, id_cl.d, win, [], [], 1/Ts);
```

The dynamics from driving voltage to the measured displacement are compared both in the open-loop and IFF case, and for the test-bench experimental identification and for the Simscape model in Figure 2.6. This shows that the Integral Force Feedback architecture effectively damps the first resonance of the system.

2.6 Inertial Sensors

In order to estimate the dynamics of the inertial sensor (the transfer function from the “absolute” displacement to the measured voltage), the following experiment can be performed:

- The mass is excited such that its relative displacement as measured by the interferometer is much larger than the ground “absolute” motion.
- The transfer function from the measured displacement by the interferometer to the measured voltage generated by the inertial sensors can be estimated.

The first point is quite important in order to have a good coherence between the interferometer measurement and the inertial sensor measurement.

Here, a first identification is performed where the excitation signal is a white noise.

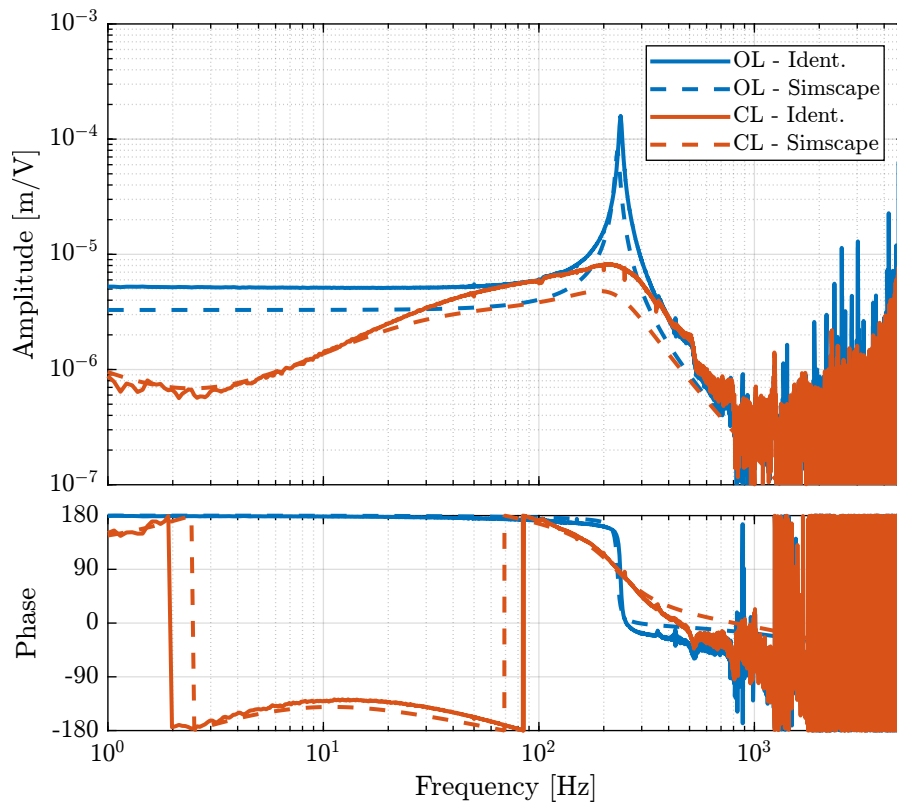


Figure 2.6: Comparison of the open-loop and closed-loop (IFF) dynamics for both the real identification and the Simscape one

As usual, the data is loaded and any offset is removed.

```
Matlab
id = load('identification_noise_opt_iff.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');

id.d = detrend(id.d, 0);
id.acc_1 = detrend(id.acc_1, 0);
id.acc_2 = detrend(id.acc_2, 0);
id.geo_1 = detrend(id.geo_1, 0);
id.geo_2 = detrend(id.geo_2, 0);
id.f_meas = detrend(id.f_meas, 0);
```

Then the transfer functions from the measured displacement by the interferometer to the generated voltage of the inertial sensors are computed..

```
Matlab
Ts = id.t(2) - id.t(1);
win = hann(ceil(10/Ts));
```

```
Matlab
[tf_acc1_est, f] = tfestimate(id.d, id.acc_1, win, [], [], 1/Ts);
[co_acc1_est, ~] = mscohere( id.d, id.acc_1, win, [], [], 1/Ts);
[tf_acc2_est, ~] = tfestimate(id.d, id.acc_2, win, [], [], 1/Ts);
[co_acc2_est, ~] = mscohere( id.d, id.acc_2, win, [], [], 1/Ts);

[tf_geo1_est, ~] = tfestimate(id.d, id.geo_1, win, [], [], 1/Ts);
[co_geo1_est, ~] = mscohere( id.d, id.geo_1, win, [], [], 1/Ts);
[tf_geo2_est, ~] = tfestimate(id.d, id.geo_2, win, [], [], 1/Ts);
[co_geo2_est, ~] = mscohere( id.d, id.geo_2, win, [], [], 1/Ts);
```

The same transfer functions are estimated using the Simscape model.

The obtained dynamics of the accelerometer are compared in Figure 2.7 while the one of the geophones are compared in Figure 2.8.

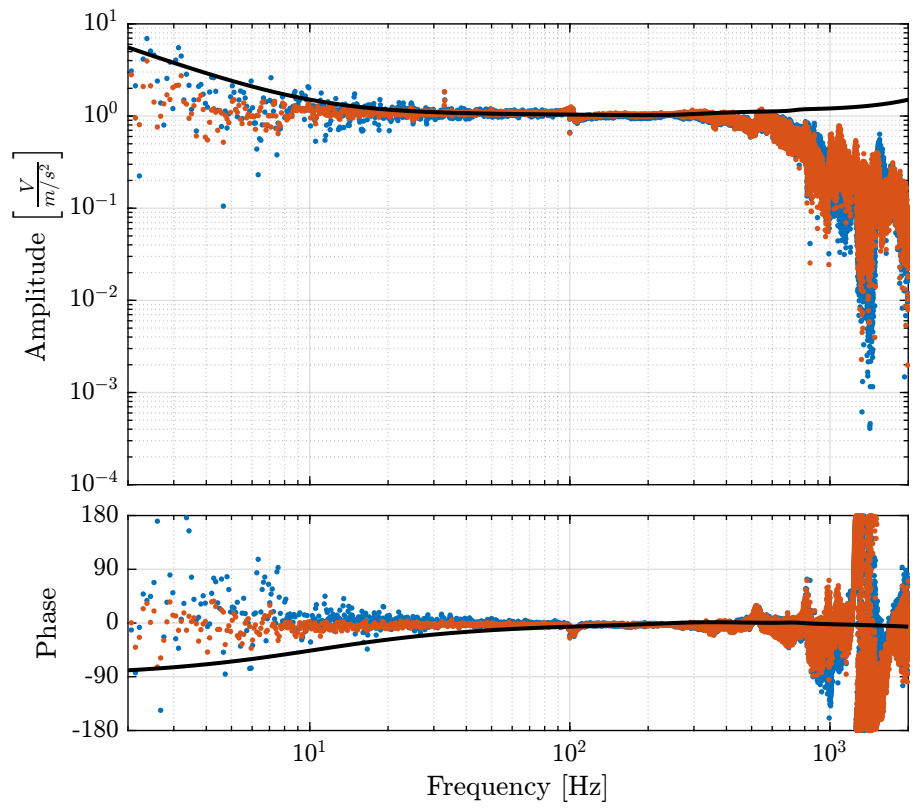


Figure 2.7: Comparison of the measured accelerometer dynamics

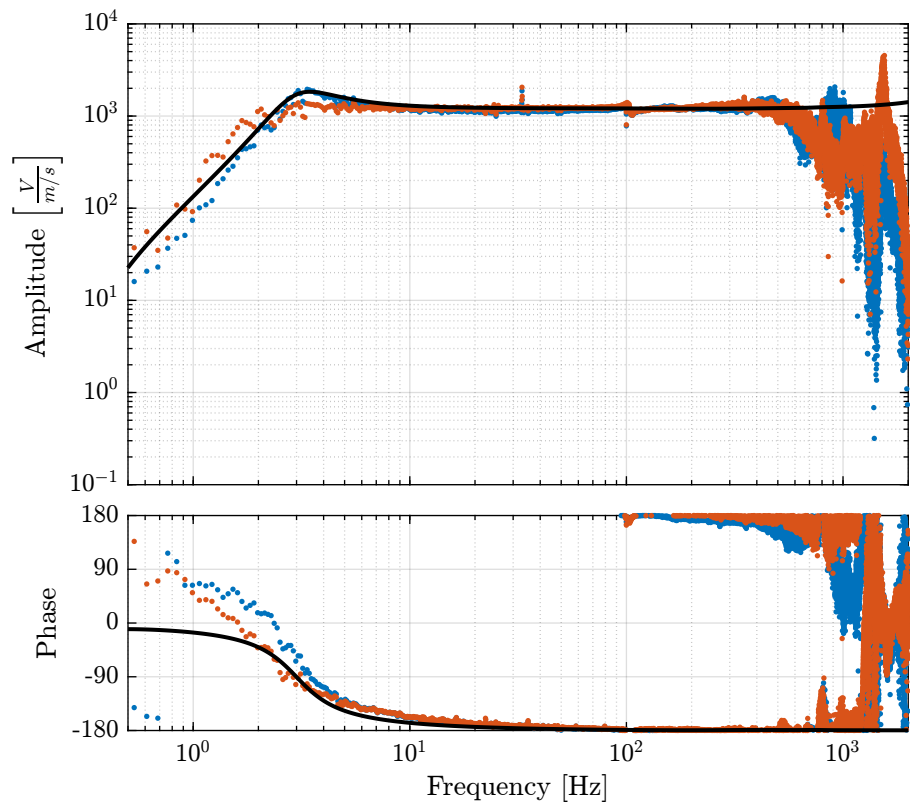


Figure 2.8: Comparison of the measured geophone dynamics

3 Optimal IFF Development

In this section, a proper identification of the transfer function from the force actuator to the force sensor is performed. Then, an optimal IFF controller is developed and applied experimentally.

The damped system is identified to verify the effectiveness of the added method.

3.1 Load Data

The experimental data is loaded and any offset is removed.

```
id_ol = load('identification_noise_bis.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
id_ol.d = detrend(id_ol.d, 0);  
id_ol.acc_1 = detrend(id_ol.acc_1, 0);  
id_ol.acc_2 = detrend(id_ol.acc_2, 0);  
id_ol.geo_1 = detrend(id_ol.geo_1, 0);  
id_ol.geo_2 = detrend(id_ol.geo_2, 0);  
id_ol.f_meas = detrend(id_ol.f_meas, 0);  
id_ol.u = detrend(id_ol.u, 0);
```

3.2 Experimental Data

The transfer function from force actuator to force sensors is estimated.

The coherence shown in Figure 3.1 shows that the excitation signal is good enough.

```
Ts = id_ol.t(2) - id_ol.t(1);  
win = hann(ceil(10/Ts));
```

```
[tf_fmeas_est, f] = tfestimate(id_ol.u, id_ol.f_meas, win, [], [], 1/Ts); % [V/m]  
[co_fmeas_est, ~] = mscohere(id_ol.u, id_ol.f_meas, win, [], [], 1/Ts);
```

The obtained dynamics is shown in Figure 3.2.

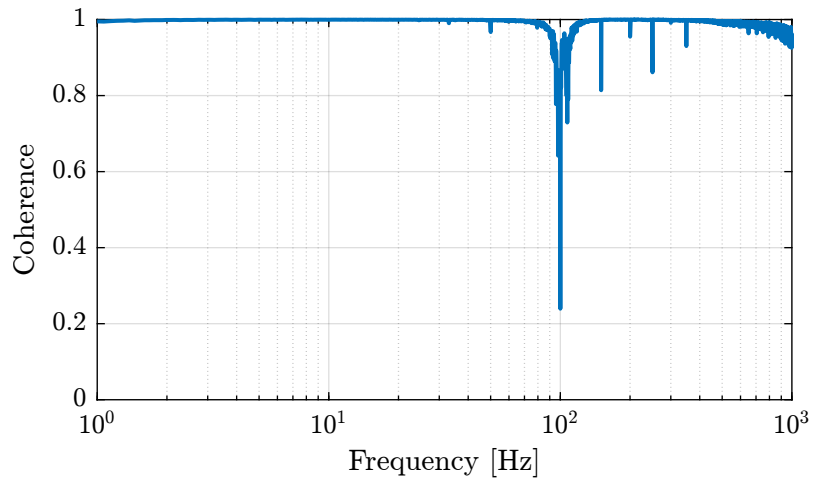


Figure 3.1: Coherence for the identification of the IFF plant

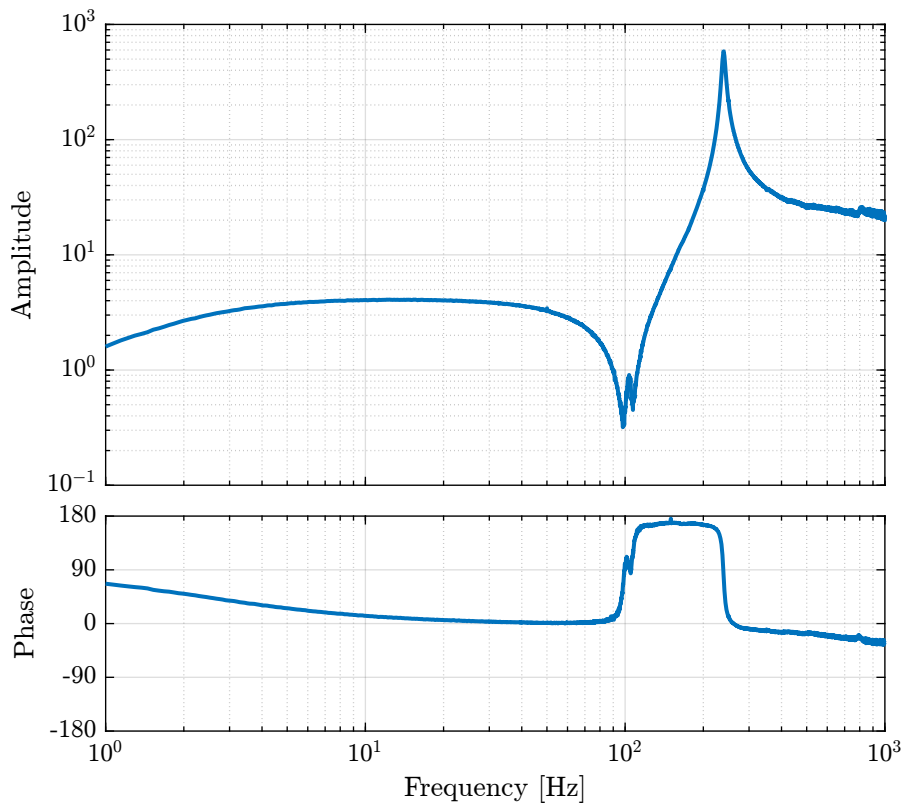


Figure 3.2: Bode plot of the identified IFF plant

3.3 Model of the IFF Plant

In order to plot the root locus for the IFF control strategy, a model of the identified plant is developed.

It consists of several poles and zeros are shown below.

```
Matlab
wz = 2*pi*102;
xi_z = 0.01;
wp = 2*pi*239.4;
xi_p = 0.015;

Giff = 2.2*(s^2 + 2*xi_z*s*wz + wz^2)/(s^2 + 2*xi_p*s*wp + wp^2) * ... % Dynamics
10*(s/3/pi/(1 + s/3/pi)) * ... % Low pass filter and gain of the voltage amplifier
exp(-Ts*s); % Time delay induced by ADC/DAC
```

The comparison of the identified dynamics and the developed model is done in Figure 3.3.

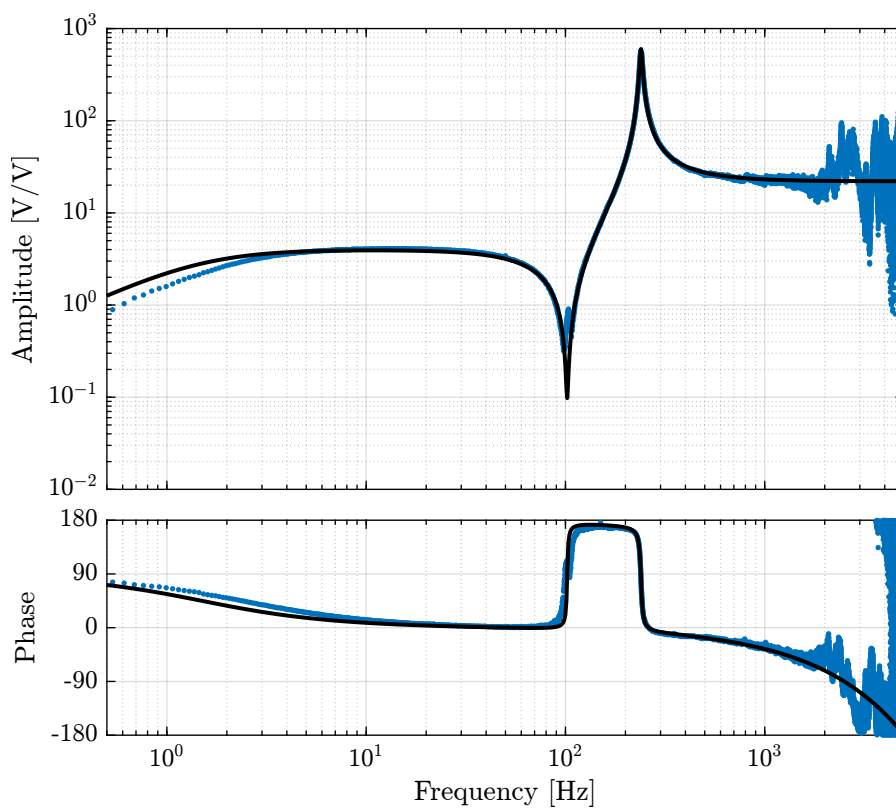


Figure 3.3: IFF Plant + Model

3.4 Root Locus and optimal Controller

Now, the root locus for the Integral Force Feedback strategy is computed and shown in Figure 3.4.

Note that the controller used is not a pure integrator but rather a first order low pass filter with a cut-off frequency set at 2Hz.

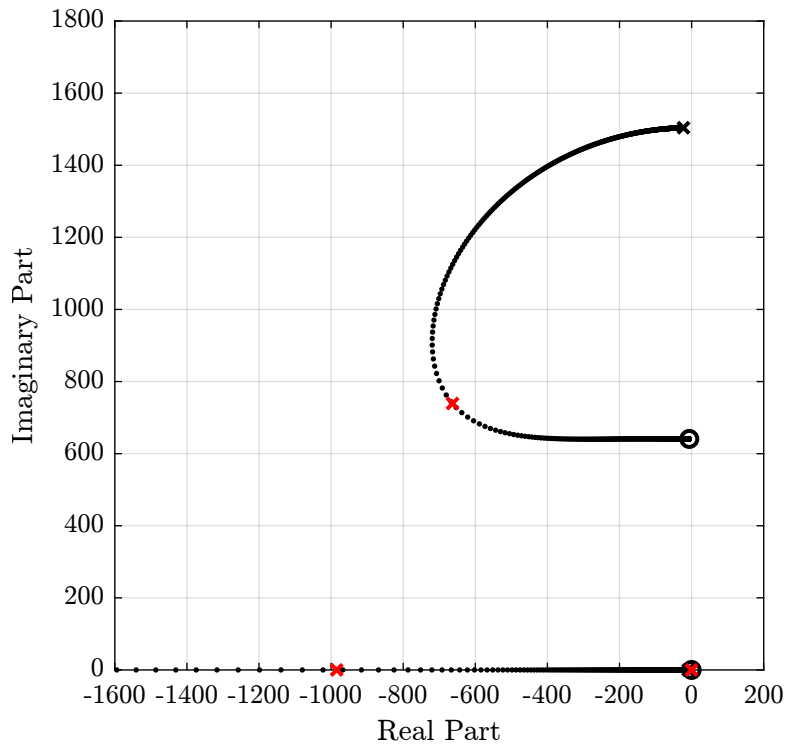


Figure 3.4: Root Locus for the IFF control

The controller that yield maximum damping (shown by the red cross in Figure 3.4) is:

```
Matlab
Kiff_opt = 102/(s + 2*pi*2);
```

3.5 Verification of the achievable damping

A new identification is performed with the IFF control strategy applied to the system.

Data is loaded and offset removed.

```
Matlab
id_cl = load('identification_noise_iff_bis.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
Matlab
id_cl.d = detrend(id_cl.d, 0);
id_cl.acc_1 = detrend(id_cl.acc_1, 0);
id_cl.acc_2 = detrend(id_cl.acc_2, 0);
id_cl.geo_1 = detrend(id_cl.geo_1, 0);
id_cl.geo_2 = detrend(id_cl.geo_2, 0);
id_cl.f_meas = detrend(id_cl.f_meas, 0);
id_cl.u = detrend(id_cl.u, 0);
```

The open-loop and closed-loop dynamics are estimated.

```
Matlab
[tf_G_ol_est, f] = tfestimate(id_ol.u, id_ol.d, win, [], [], 1/Ts);
[co_G_ol_est, ~] = mscohere( id_ol.u, id_ol.d, win, [], [], 1/Ts);
[tf_G_cl_est, ~] = tfestimate(id_cl.u, id_cl.d, win, [], [], 1/Ts);
[co_G_cl_est, ~] = mscohere( id_cl.u, id_cl.d, win, [], [], 1/Ts);
```

The obtained coherence is shown in Figure 3.5 and the dynamics in Figure 3.6.

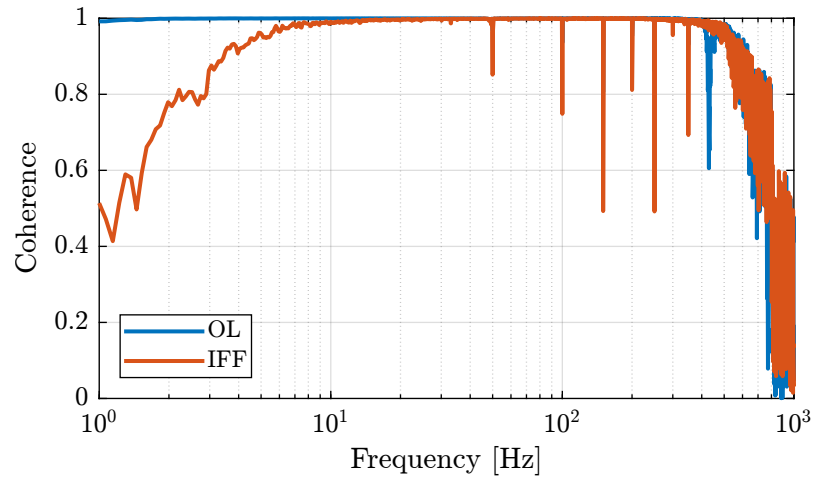


Figure 3.5: Coherence for the transfer function from F to d, with and without IFF

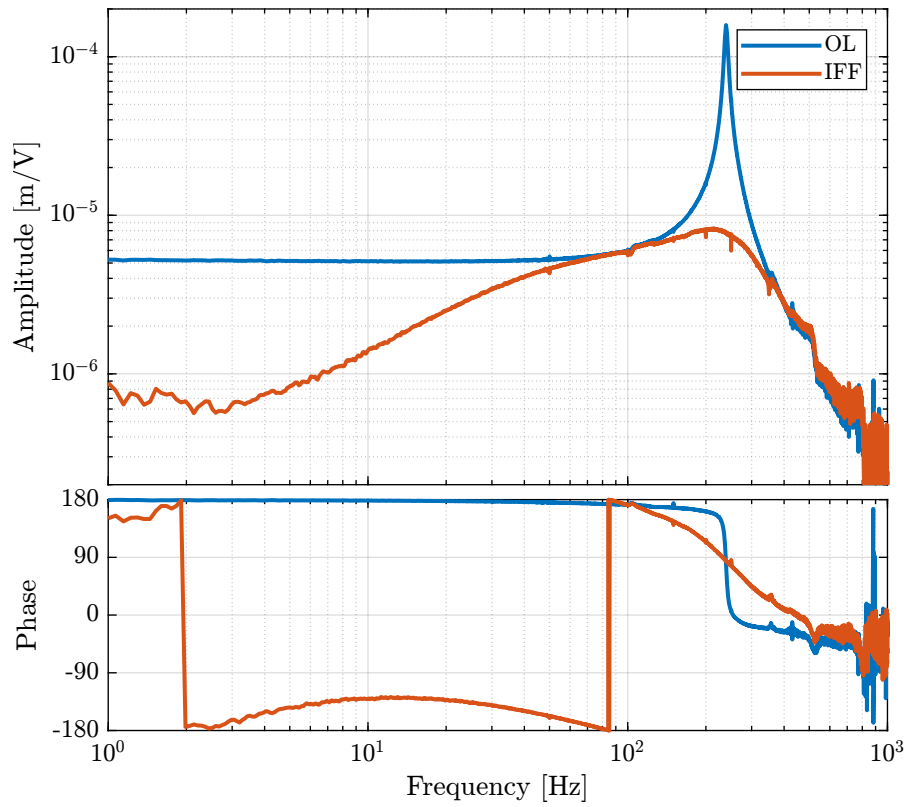


Figure 3.6: Coherence for the transfer function from F to d, with and without IFF

4 Generate the excitation signal

In order to properly estimate the dynamics of the inertial sensor, the excitation signal must be properly chosen.

The requirements on the excitation signal is:

- General much larger motion than the measured motion during the huddle test
- Don't damage the actuator

To determine the perfect voltage signal to be generated, we need two things:

- the transfer function from voltage to mass displacement
- the PSD of the measured motion by the inertial sensors
- not saturate the sensor signals
- provide enough signal/noise ratio (good coherence) in the frequency band of interest ($\sim 0.5\text{Hz}$ to 3kHz)

4.1 Transfer function from excitation signal to displacement

Let's first estimate the transfer function from the excitation signal in [V] to the generated displacement in [m] as measured by the interferometer.

```
Matlab
id_cl = load('identification_noise_iff_bis.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
Matlab
Ts = id_cl.t(2) - id_cl.t(1);
win = hann(ceil(10/Ts));
```

```
Matlab
[tf_G_cl_est, f] = tfestimate(id_cl.u, id_cl.d, win, [], [], 1/Ts);
[co_G_cl_est, ~] = mscohere(id_cl.u, id_cl.d, win, [], [], 1/Ts);
```

Approximate transfer function from voltage output to generated displacement when IFF is used, in [m/V].


```

Matlab
G_d_est = -5e-6*(2*pi*230)^2/(s^2 + 2*0.3*2*pi*240*s + (2*pi*240)^2);

```

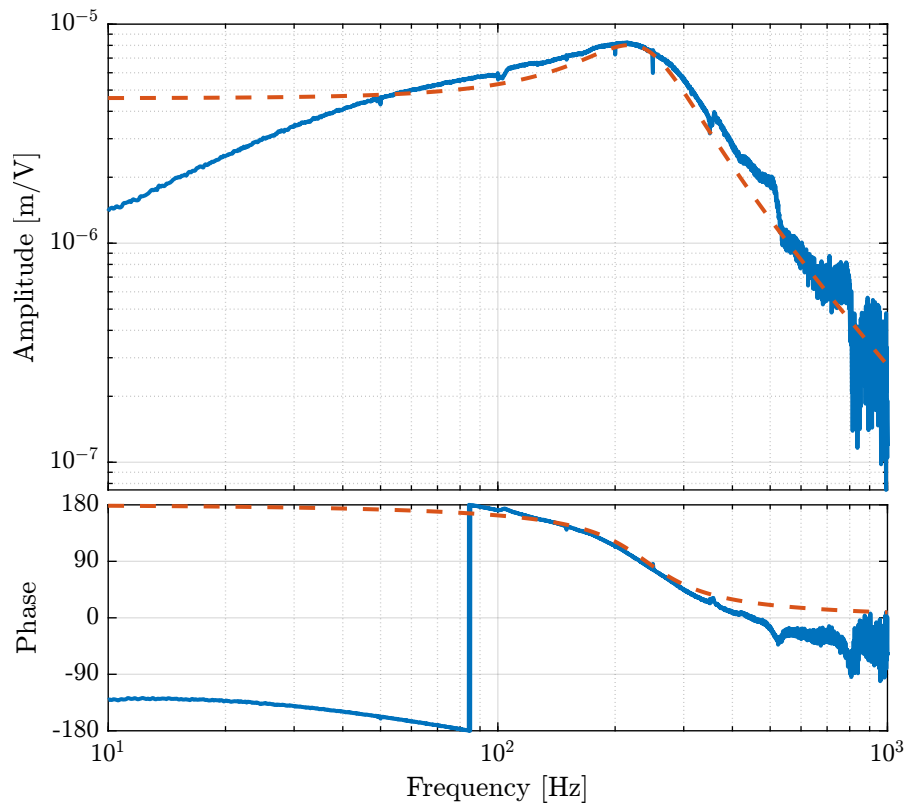


Figure 4.1: Estimation of the transfer function from the excitation signal to the generated displacement

4.2 Motion measured during Huddle test

We now compute the PSD of the measured motion by the inertial sensors during the huddle test.

```

Matlab
ht = load('huddle_test.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
ht.d = detrend(ht.d, 0);
ht.acc_1 = detrend(ht.acc_1, 0);
ht.acc_2 = detrend(ht.acc_2, 0);
ht.geo_1 = detrend(ht.geo_1, 0);
ht.geo_2 = detrend(ht.geo_2, 0);

```

```

Matlab
[p_d, f] = pwelch(ht.d, win, [], [], 1/Ts);
[p_acc1, ~] = pwelch(ht.acc_1, win, [], [], 1/Ts);
[p_acc2, ~] = pwelch(ht.acc_2, win, [], [], 1/Ts);
[p_geo1, ~] = pwelch(ht.geo_1, win, [], [], 1/Ts);
[p_geo2, ~] = pwelch(ht.geo_2, win, [], [], 1/Ts);

```

Using an estimated model of the sensor dynamics from the documentation of the sensors, we can compute the ASD of the motion in m/\sqrt{Hz} measured by the sensors.

```

Matlab
G_acc = 1/(1 + s/2/pi/2500); % [V/(m/s^2)]
G_geo = -120*s^2/(s^2 + 2*0.7*2*pi*2*s + (2*pi*2)^2); % [V/(m/s)]

```

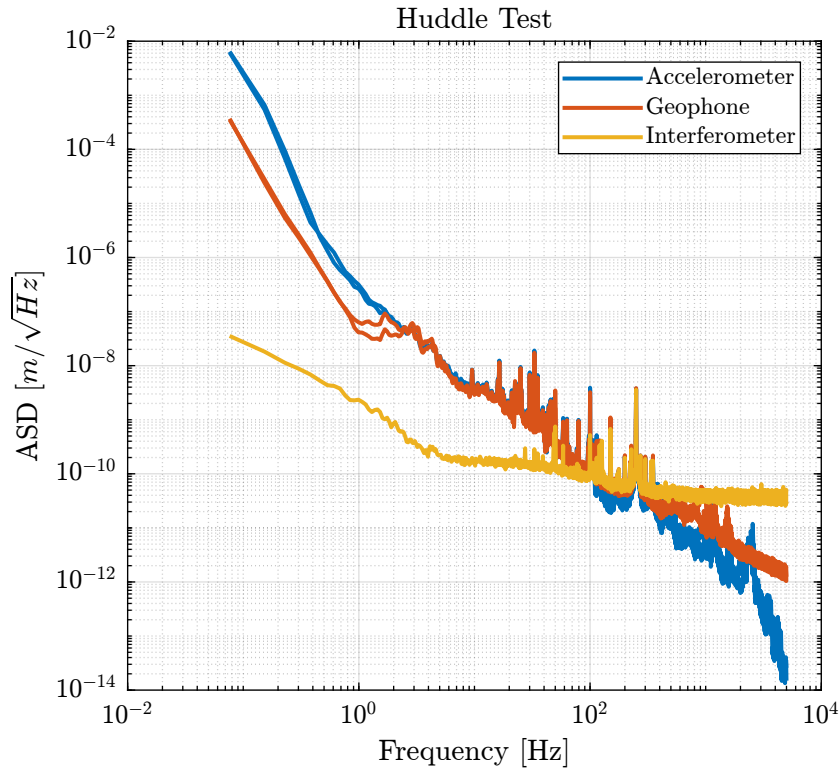


Figure 4.2: ASD of the motion measured by the sensors

From the ASD of the motion measured by the sensors, we can create an excitation signal that will generate much motion motion that the motion under no excitation.

We create `G_exc` that corresponds to the wanted generated motion.

```

Matlab
G_exc = 0.2e-6/(1 + s/2/pi/2)/(1 + s/2/pi/50);

```

And we create a time domain signal `y_d` that have the spectral density described by `G_exc`.

```

Matlab
Fs = 1/Ts;
t = 0:Ts:180; % Time Vector [s]
u = sqrt(Fs/2)*randn(length(t), 1); % Signal with an ASD equal to one
y_d = lsim(G_exc, u, t);

```

```
[pxx, ~] = pwelch(y_d, win, 0, [], Fs);
```

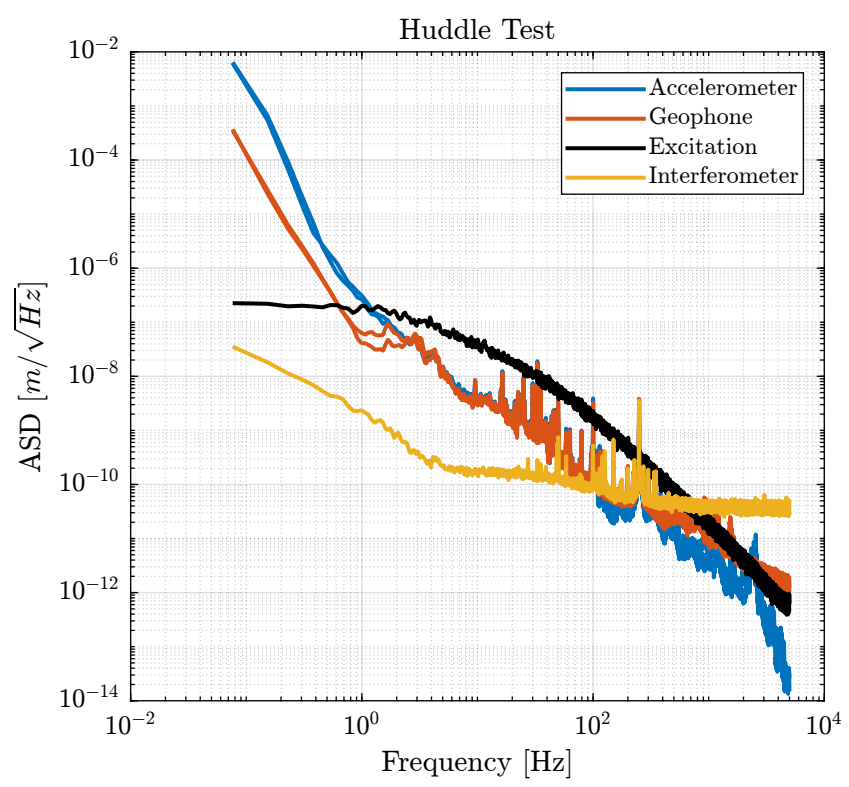


Figure 4.3: Comparison of the ASD of the motion during Huddle and the wanted generated motion

We can now generate the voltage signal that will generate the wanted motion.

```
y_v = lsim(G_exc * ... % from unit PSD to shaped PSD  
(1 + s/2/pi/50) * ... % Inverse of pre-filter included in the Simulink file  
1/G_d_est * ... % Wanted displacement => required voltage  
1/(1 + s/2/pi/5e3), ... % Add some high frequency filtering  
u, t);
```

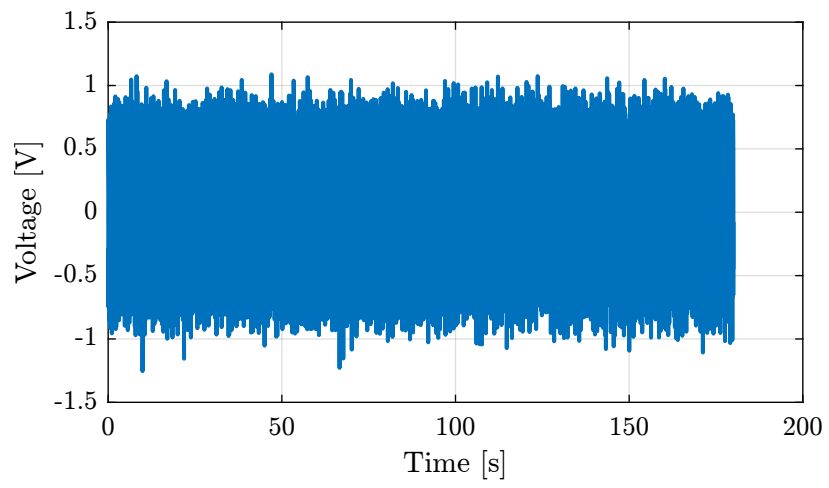


Figure 4.4: Generated excitation signal

5 Identification of the Inertial Sensors Dynamics

Using the excitation signal generated in Section 4, the dynamics of the inertial sensors are identified.

5.1 Load Data

Both the measurement data during the identification test and during an “huddle test” are loaded.

```
Matlab
id = load('identification_noise_opt_iff.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
ht = load('huddle_test.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
Matlab
ht.d = detrend(ht.d, 0);
ht.acc_1 = detrend(ht.acc_1, 0);
ht.acc_2 = detrend(ht.acc_2, 0);
ht.geo_1 = detrend(ht.geo_1, 0);
ht.geo_2 = detrend(ht.geo_2, 0);
ht.f_meas = detrend(ht.f_meas, 0);
```

```
Matlab
id.d = detrend(id.d, 0);
id.acc_1 = detrend(id.acc_1, 0);
id.acc_2 = detrend(id.acc_2, 0);
id.geo_1 = detrend(id.geo_1, 0);
id.geo_2 = detrend(id.geo_2, 0);
id.f_meas = detrend(id.f_meas, 0);
```

5.2 Compare PSD during Huddle and and during identification

The Power Spectral Density of the measured motion during the huddle test and during the identification test are compared in Figures 5.1 and 5.2.

```
Matlab
Ts = ht.t(2) - ht.t(1);
win = hann(ceil(10/Ts));
```

```

Matlab
[p_id_d, f] = pwelch(id.d, win, [], [], 1/Ts);
[p_id_acc1, ~] = pwelch(id.acc_1, win, [], [], 1/Ts);
[p_id_acc2, ~] = pwelch(id.acc_2, win, [], [], 1/Ts);
[p_id_geo1, ~] = pwelch(id.geo_1, win, [], [], 1/Ts);
[p_id_geo2, ~] = pwelch(id.geo_2, win, [], [], 1/Ts);

```

```

Matlab
[p_ht_d, ~] = pwelch(ht.d, win, [], [], 1/Ts);
[p_ht_acc1, ~] = pwelch(ht.acc_1, win, [], [], 1/Ts);
[p_ht_acc2, ~] = pwelch(ht.acc_2, win, [], [], 1/Ts);
[p_ht_geo1, ~] = pwelch(ht.geo_1, win, [], [], 1/Ts);
[p_ht_geo2, ~] = pwelch(ht.geo_2, win, [], [], 1/Ts);
[p_ht_fmeas, ~] = pwelch(ht.f_meas, win, [], [], 1/Ts);

```

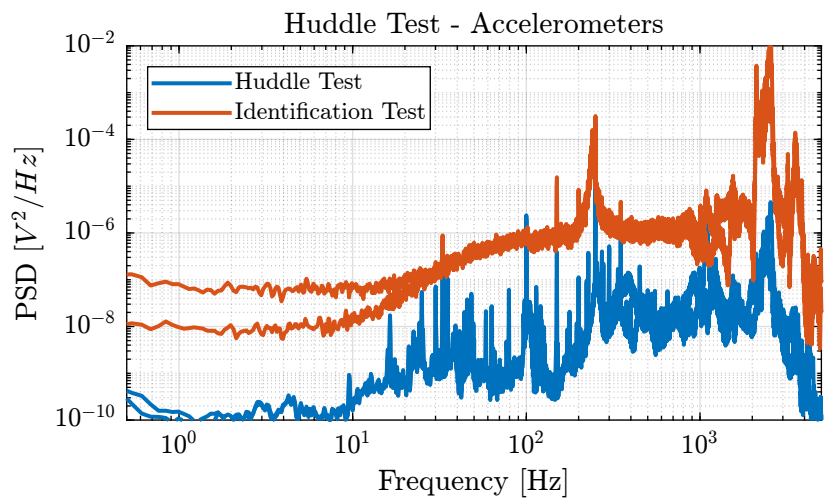


Figure 5.1: Comparison of the PSD of the measured motion during the Huddle test and during the identification

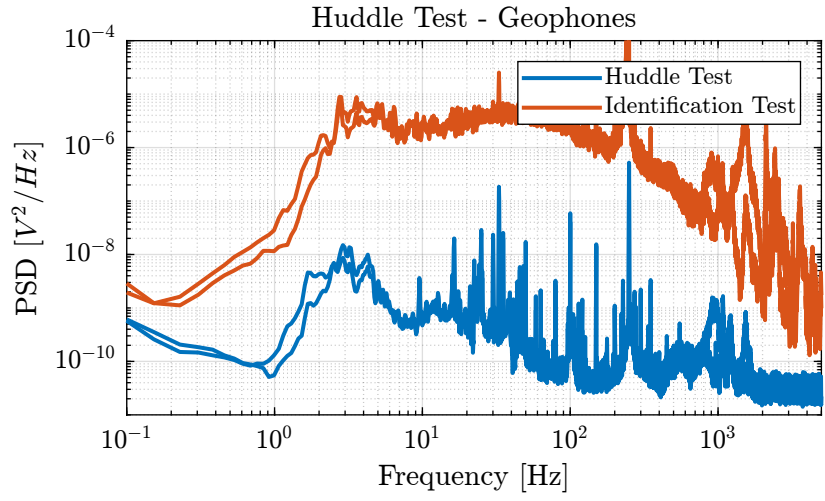


Figure 5.2: Comparison of the PSD of the measured motion during the Huddle test and during the identification

5.3 Compute transfer functions

The transfer functions from the motion as measured by the interferometer (and that should represent the absolute motion of the mass) to the inertial sensors are estimated:

```
Matlab
[tf_acc1_est, f] = tfestimate(id.d, id.acc_1, win, [], [], 1/Ts);
[co_acc1_est, ~] = mscohere( id.d, id.acc_1, win, [], [], 1/Ts);
[tf_acc2_est, ~] = tfestimate(id.d, id.acc_2, win, [], [], 1/Ts);
[co_acc2_est, ~] = mscohere( id.d, id.acc_2, win, [], [], 1/Ts);

[tf_geo1_est, ~] = tfestimate(id.d, id.geo_1, win, [], [], 1/Ts);
[co_geo1_est, ~] = mscohere( id.d, id.geo_1, win, [], [], 1/Ts);
[tf_geo2_est, ~] = tfestimate(id.d, id.geo_2, win, [], [], 1/Ts);
[co_geo2_est, ~] = mscohere( id.d, id.geo_2, win, [], [], 1/Ts);
```

The obtained coherence are shown in Figure 5.3.

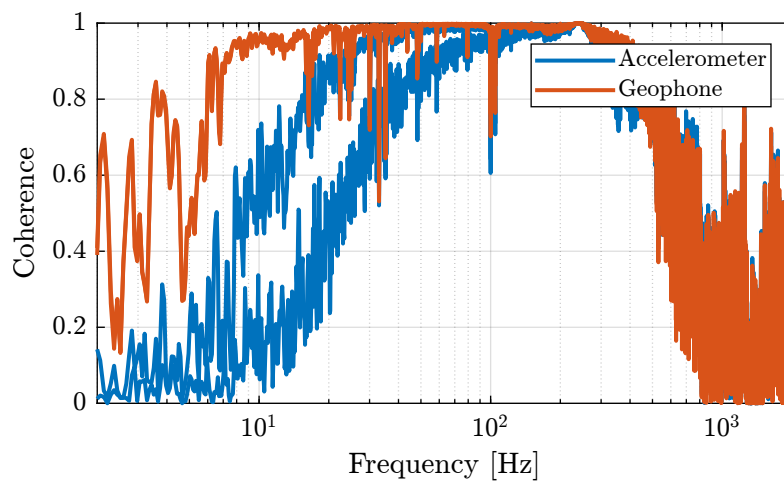


Figure 5.3: Coherence for the estimation of the sensor dynamics

We also make a simplified model of the inertial sensors to be compared with the identified dynamics.

```
Matlab
G_acc = 1/(1 + s/2/pi/2500); % [V/(m/s^2)]
G_geo = -1200*s^2/(s^2 + 2*0.7*2*pi*s + (2*pi*2)^2); % [[V/(m/s)]
```

The model and identified dynamics show good agreement (Figures 5.4 and 5.5.)

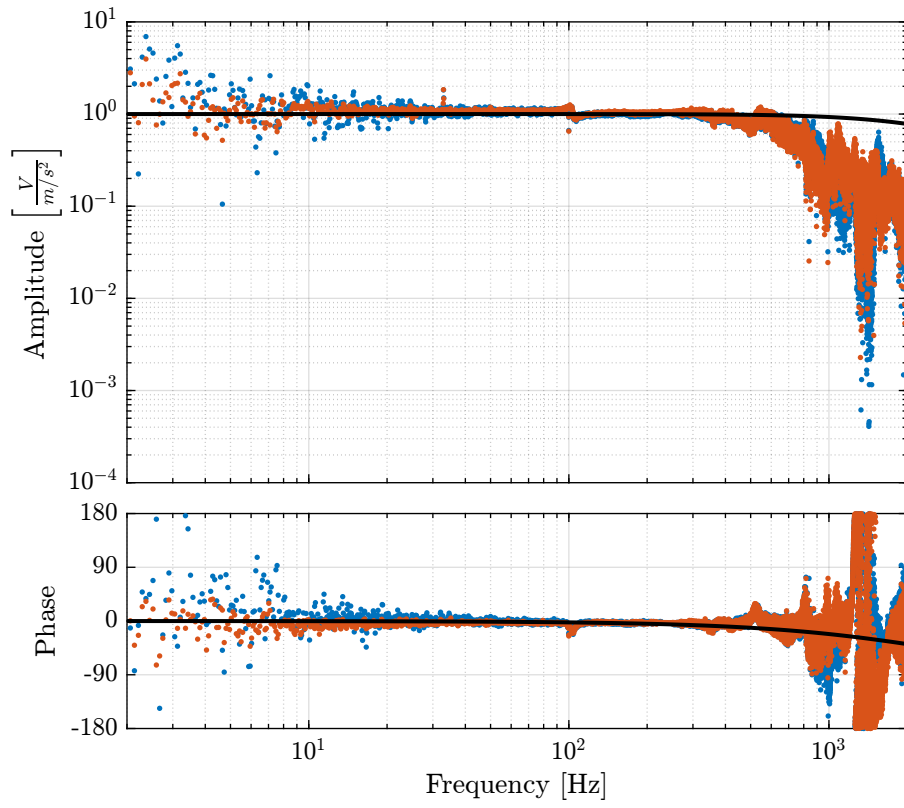


Figure 5.4: Identified dynamics of the accelerometers

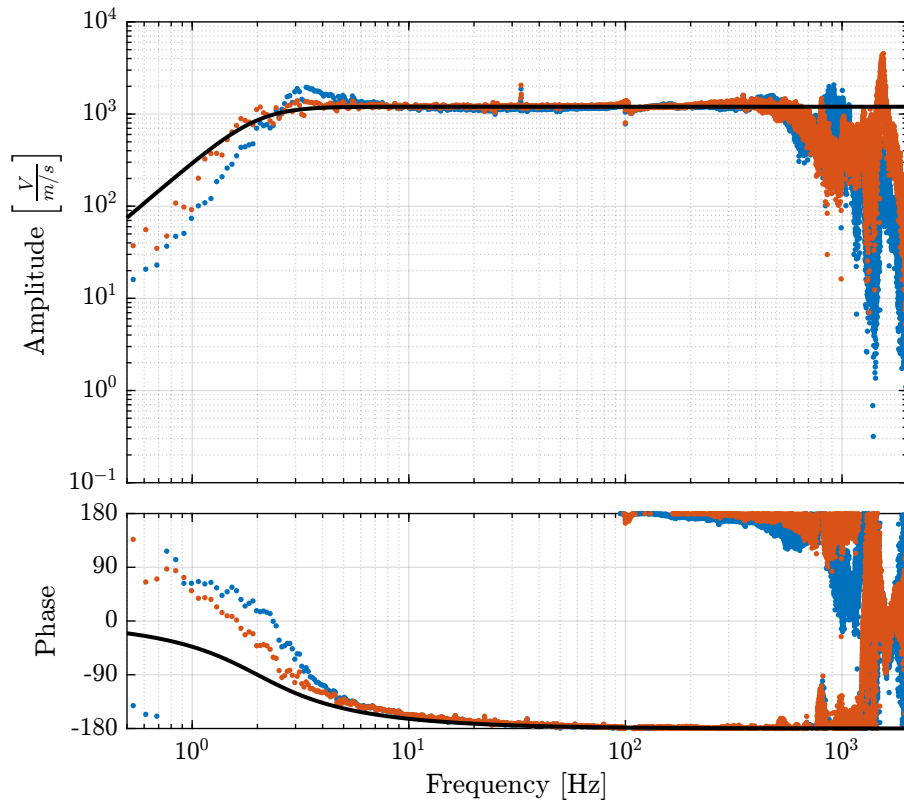


Figure 5.5: Identified dynamics of the geophones

6 Inertial Sensor Noise and the \mathcal{H}_2 Synthesis of complementary filters

In this section, the noise of the inertial sensors (geophones and accelerometers) is estimated.

6.1 Load Data

As before, the identification data is loaded and any offset is removed.

```
id = load('identification_noise_opt_iff.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
id.d = detrend(id.d, 0);  
id.acc_1 = detrend(id.acc_1, 0);  
id.acc_2 = detrend(id.acc_2, 0);  
id.geo_1 = detrend(id.geo_1, 0);  
id.geo_2 = detrend(id.geo_2, 0);  
id.f_meas = detrend(id.f_meas, 0);
```

6.2 ASD of the Measured displacement

The Power Spectral Density of the displacement as measured by the interferometer and the inertial sensors is computed.

```
Ts = id.t(2) - id.t(1);  
win = hann(ceil(10/Ts));
```

```
[p_id_d, f] = pwelch(id.d, win, [], [], 1/Ts);  
[p_id_acc1, ~] = pwelch(id.acc_1, win, [], [], 1/Ts);  
[p_id_acc2, ~] = pwelch(id.acc_2, win, [], [], 1/Ts);  
[p_id_geo1, ~] = pwelch(id.geo_1, win, [], [], 1/Ts);  
[p_id_geo2, ~] = pwelch(id.geo_2, win, [], [], 1/Ts);
```

Let's use a model of the accelerometer and geophone to compute the motion from the measured voltage.

```

G_acc = 1/(1 + s/2/pi/2500); % [V/(m/s^2)]
G_geo = -1200*s^2/(s^2 + 2*0.7*2*pi*s + (2*pi*2)^2); % [[V/(m/s)]

```

The obtained ASD in m/\sqrt{Hz} is shown in Figure 6.1.

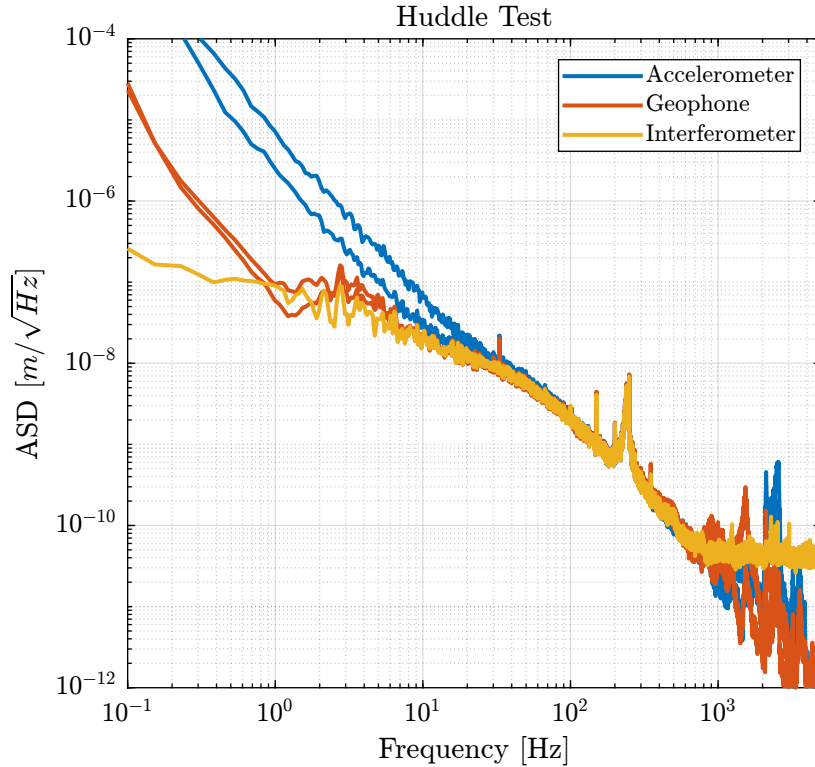


Figure 6.1: ASD of the measured displacement as measured by all the sensors

6.3 ASD of the Sensor Noise

The noise of a sensor can be estimated using two identical sensors by computing:

- the Power Spectral Density of the measured motion by the two sensors
- the Cross Spectral Density between the two sensors (coherence)

This technique to estimate the sensor noise is described in [1].

The Power Spectral Density of the sensor noise can be estimated using the following equation:

$$|S_n(\omega)| = |S_{x_1}(\omega)| \left(1 - \gamma_{x_1 x_2}(\omega)\right) \quad (6.1)$$

with S_{x_1} the PSD of one of the sensor and $\gamma_{x_1 x_2}$ the coherence between the two sensors.

The coherence between the two accelerometers and between the two geophones is computed.

```

Matlab
[coh_acc, ~] = mscohere(id.acc_1, id.acc_2, win, [], [], 1/Ts);
[coh_geo, ~] = mscohere(id.geo_1, id.geo_2, win, [], [], 1/Ts);

```

Finally, the Power Spectral Density of the sensors is computed and converted in $[m^2/Hz]$.

```

Matlab
pN_acc = p_id_acc1.*(1 - coh_acc) .* ... % [V^2/Hz]
1./abs(squeeze(freqresp(G_acc*s^2, f, 'Hz'))).^2; % [(m/V)^2]
pN_geo = p_id_geo1.*(1 - coh_geo) .* ... % [V^2/Hz]
1./abs(squeeze(freqresp(G_geo*s, f, 'Hz'))).^2; % [(m/V)^2]

```

The ASD of obtained noises are compared with the ASD of the measured signals in Figure 6.2.

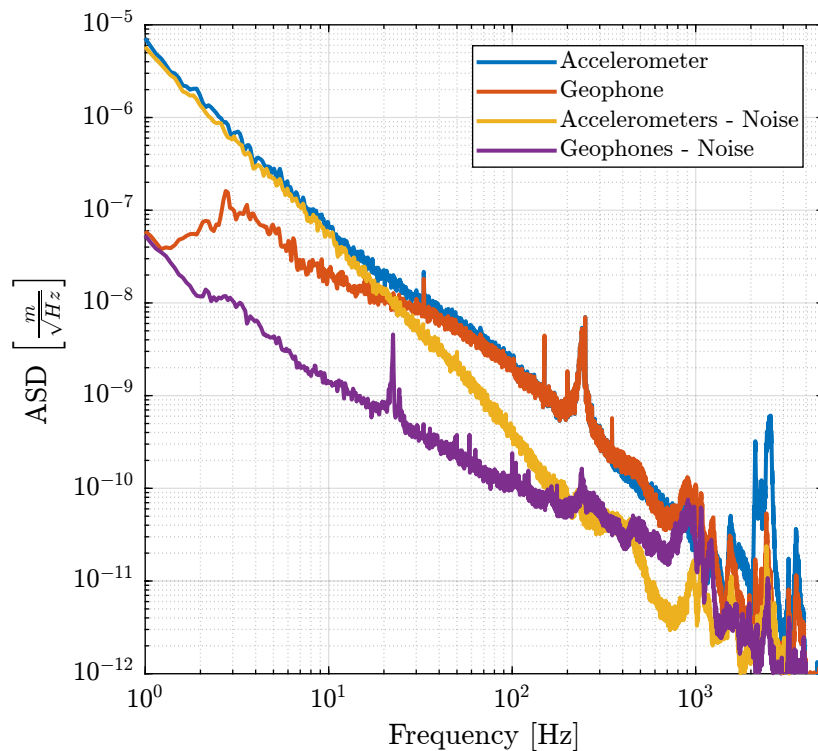


Figure 6.2: Comparison of the computed ASD of the noise of the two inertial sensors

6.4 Noise Model

Transfer functions are adjusted in order to fit the ASD of the sensor noises (expressed in $[m/s/\sqrt{Hz}]$ for more easy fitting).

These transfer functions are defined below and compared with the measured ASD in Figure 6.3.

```

Matlab
N_acc = 1*(s/(2*pi*2000) + 1)^2/(s + 0.1*2*pi)/(s + 1e3*2*pi); % [m/sqrt(Hz)]
N_geo = 4e-4*(s/(2*pi*200) + 1)/(s + 1e3*2*pi); % [m/sqrt(Hz)]

```

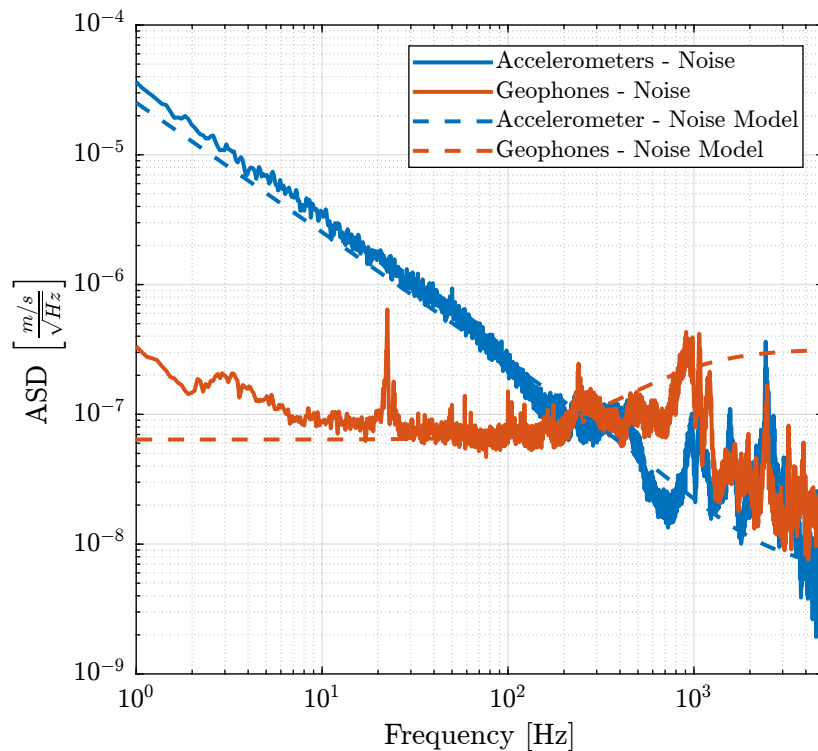


Figure 6.3: ASD of the velocity noise measured by the sensors and the noise models

6.5 \mathcal{H}_2 Synthesis of the Complementary Filters

We now wish to synthesize two complementary filters to merge the geophone and the accelerometer signal in such a way that the fused signal has the lowest possible RMS noise.

To do so, we use the \mathcal{H}_2 synthesis where the transfer functions representing the noise density of both sensors are used as weights.

The generalized plant used for the synthesis is defined below.

```

Matlab
P = [0      N_acc  1;
     N_geo -N_acc  0];

```

And the \mathcal{H}_2 synthesis is done using the `h2syn` command.

```

Matlab
[H_geo, ~, gamma] = h2syn(P, 1, 1);
H_acc = 1 - H_geo;

```

The obtained complementary filters are shown in Figure 6.4.

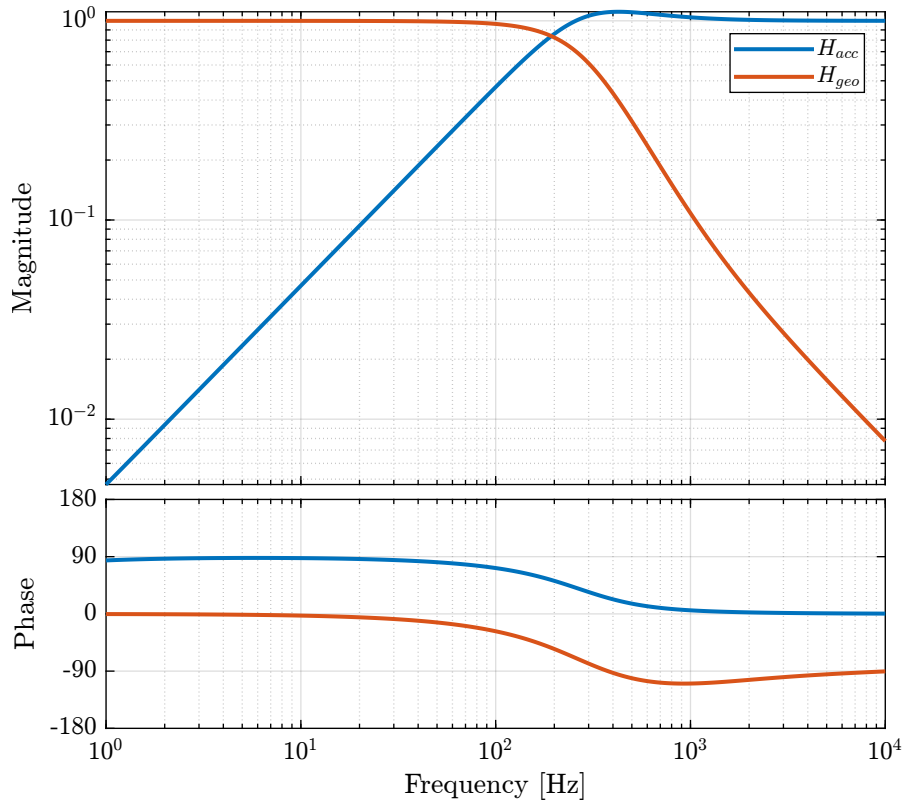


Figure 6.4: Obtained Complementary Filters

6.6 Results

Finally, the signals of both sensors are merged using the complementary filters and the super sensor noise is estimated and compared with the individual sensor noises in Figure 6.5.

Finally, the Cumulative Power Spectrum is computed and compared in Figure 6.6.

```
Matlab
[~, i_1Hz] = min(abs(f - 1));
```

```
Matlab
CPS_acc = 1/pi*flip(-cumtrapz(2*pi*flip(f), flip((pN_acc.*(2*pi*f)).^2)));
CPS_geo = 1/pi*flip(-cumtrapz(2*pi*flip(f), flip((pN_geo.*(2*pi*f)).^2)));
CPS_SS = 1/pi*flip(-cumtrapz(2*pi*flip(f), flip((pN_acc.*(2*pi*f)).^2.*abs(squeeze(freqresp(H_acc, f, 'Hz'))).^2 +
→ (pN_geo.*(2*pi*f)).^2.*abs(squeeze(freqresp(H_geo, f, 'Hz'))).^2)));
```

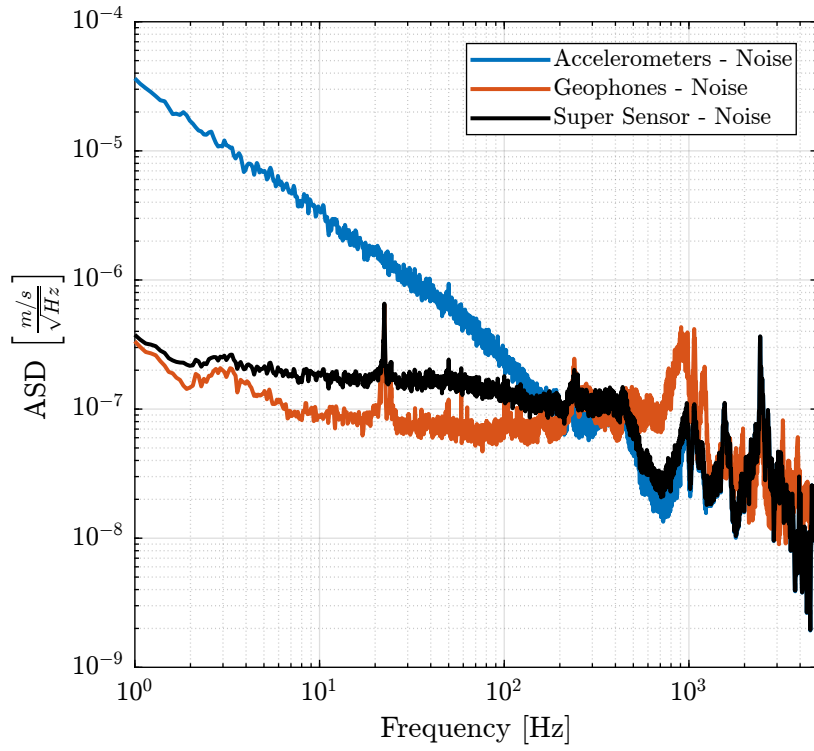


Figure 6.5: ASD of the super sensor noise (velocity)

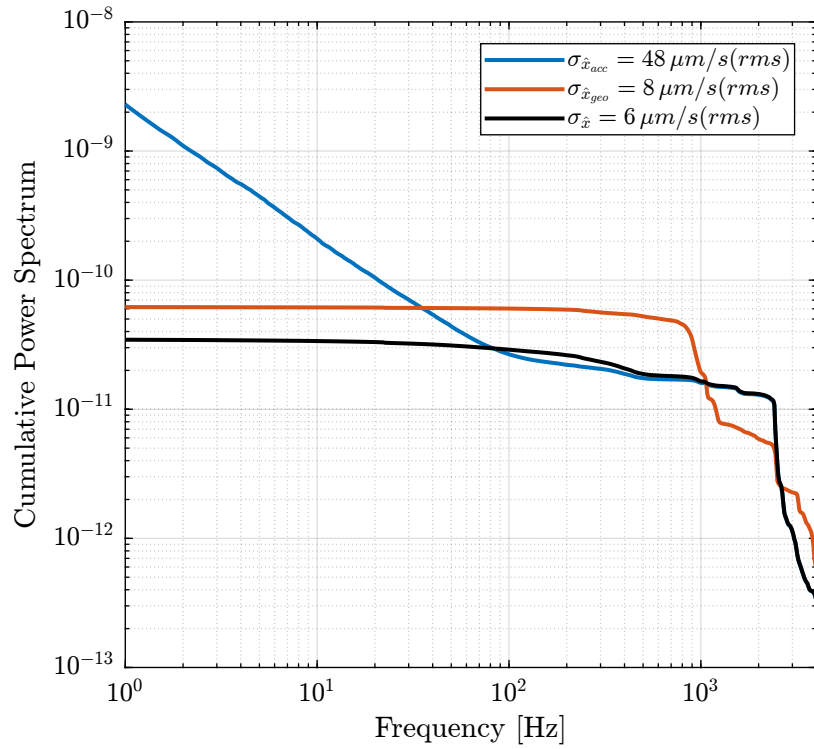


Figure 6.6: Cumulative Power Spectrum of the Sensor Noise (velocity)

7 Inertial Sensor Dynamics Uncertainty and the \mathcal{H}_∞ Synthesis of complementary filters

When merging two sensors, it is important to be sure that we correctly know the sensor dynamics near the merging frequency. Thus, identifying the uncertainty on the sensor dynamics is quite important to perform a robust merging.

7.1 Load Data

Data is loaded and offset is removed.

```
id = load('identification_noise_opt_iff.mat', 'd', 'acc_1', 'acc_2', 'geo_1', 'geo_2', 'f_meas', 'u', 't');
```

```
id.d = detrend(id.d, 0);  
id.acc_1 = detrend(id.acc_1, 0);  
id.acc_2 = detrend(id.acc_2, 0);  
id.geo_1 = detrend(id.geo_1, 0);  
id.geo_2 = detrend(id.geo_2, 0);  
id.f_meas = detrend(id.f_meas, 0);
```

7.2 Compute the dynamics of both sensors

The dynamics of inertial sensors are estimated (in $[V/m]$).

```
Ts = id.t(2) - id.t(1);  
win = hann(ceil(10/Ts));
```

```
[tf_acc1_est, f] = tfestimate(id.d, id.acc_1, win, [], [], 1/Ts);  
[co_acc1_est, ~] = mscohere( id.d, id.acc_1, win, [], [], 1/Ts);  
[tf_acc2_est, ~] = tfestimate(id.d, id.acc_2, win, [], [], 1/Ts);  
[co_acc2_est, ~] = mscohere( id.d, id.acc_2, win, [], [], 1/Ts);  
  
[tf_geo1_est, ~] = tfestimate(id.d, id.geo_1, win, [], [], 1/Ts);  
[co_geo1_est, ~] = mscohere( id.d, id.geo_1, win, [], [], 1/Ts);  
[tf_geo2_est, ~] = tfestimate(id.d, id.geo_2, win, [], [], 1/Ts);  
[co_geo2_est, ~] = mscohere( id.d, id.geo_2, win, [], [], 1/Ts);
```


The (nominal) models of the inertial sensors from the absolute displacement to the generated voltage are defined below:

```

Matlab
G_acc = 1/(1 + s/2/pi/2000)
G_geo = -1200*s^2/(s^2 + 2*0.7*2*pi*s + (2*pi)^2);

```

These models are very simplistic models, and we then take into account the un-modelled dynamics with dynamical uncertainty.

7.3 Dynamics uncertainty estimation

Weights representing the dynamical uncertainty of the sensors are defined below.

```

Matlab
w_acc = createWeight('n', 2, 'G0', 10, 'G1', 0.2, 'Gc', 1, 'w0', 6*2*pi) * ...
        createWeight('n', 2, 'G0', 1, 'G1', 5/0.2, 'Gc', 1/0.2, 'w0', 1300*2*pi);

w_geo = createWeight('n', 2, 'G0', 0.6, 'G1', 0.2, 'Gc', 0.3, 'w0', 3*2*pi) * ...
        createWeight('n', 2, 'G0', 1, 'G1', 10/0.2, 'Gc', 1/0.2, 'w0', 800*2*pi);

```

The measured dynamics are compared with the modelled one as well as the modelled uncertainty in Figure 7.1 for the accelerometers and in Figure 7.2 for the geophones.

7.4 \mathcal{H}_∞ Synthesis of Complementary Filters

A last weight is now defined that represents the maximum dynamical uncertainty that is allowed for the super sensor.

```

Matlab
wu = inv(createWeight('n', 2, 'G0', 0.7, 'G1', 0.3, 'Gc', 0.4, 'w0', 3*2*pi) * ...
         createWeight('n', 2, 'G0', 1, 'G1', 6/0.3, 'Gc', 1/0.3, 'w0', 1200*2*pi));

```

This dynamical uncertainty is compared with the two sensor uncertainties in Figure 7.3.

The generalized plant used for the synthesis is defined:

```

Matlab
P = [wu*w_acc -wu*w_acc;
     0         wu*w_geo;
     1         0];

```

And the \mathcal{H}_∞ synthesis using the `hinfsyn` command is performed.

```

Matlab
[H_geo, ~, gamma, ~] = hinfsyn(P, 1, 1, 'TOLGAM', 0.001, 'METHOD', 'ric', 'DISPLAY', 'on');

```

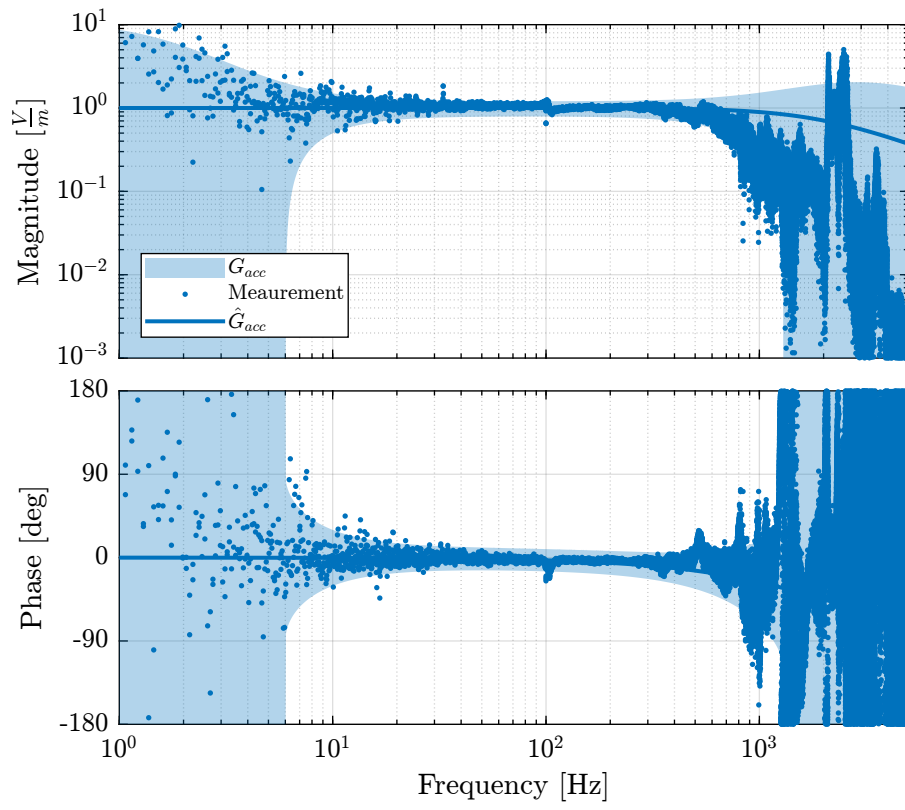


Figure 7.1: Modeled dynamical uncertainty and meared dynamics of the accelerometers

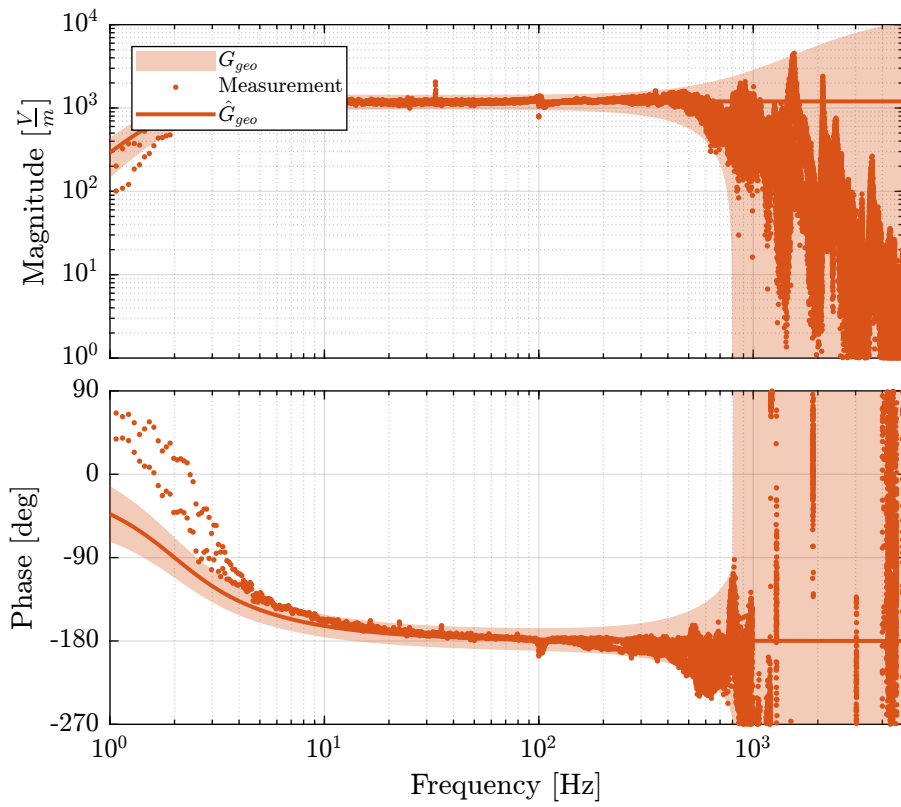


Figure 7.2: Modeled dynamical uncertainty and measured dynamics of the geophones

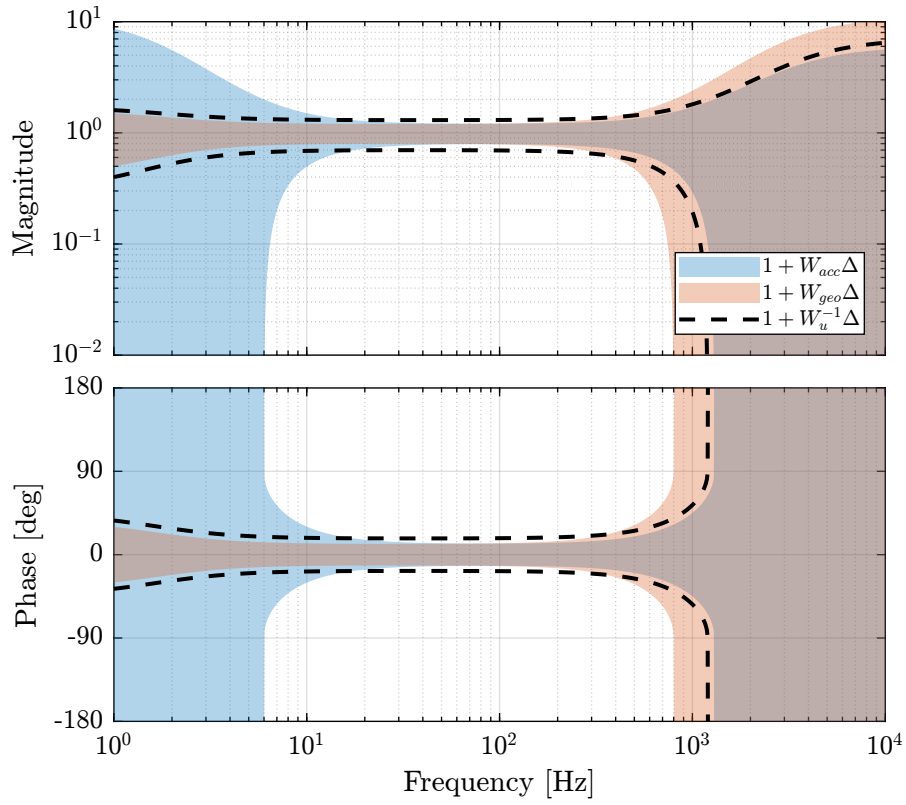


Figure 7.3: Individual sensor uncertainty (normalized by their dynamics) and the wanted maximum super sensor noise uncertainty

Results				
Test bounds: 0.8556 <= gamma <= 1.34				
gamma	X>=0	Y>=0	rho(XY)<1	p/f
1.071e+00	0.0e+00	0.0e+00	0.000e+00	p
9.571e-01	0.0e+00	0.0e+00	9.436e-16	p
9.049e-01	0.0e+00	0.0e+00	1.084e-15	p
8.799e-01	0.0e+00	0.0e+00	1.191e-16	p
8.677e-01	0.0e+00	0.0e+00	6.905e-15	p
8.616e-01	0.0e+00	0.0e+00	0.000e+00	p
8.586e-01	1.1e-17	0.0e+00	6.917e-16	p
8.571e-01	0.0e+00	0.0e+00	6.991e-17	p
8.564e-01	0.0e+00	0.0e+00	1.492e-16	p
Best performance (actual): 0.8563				

The complementary filter is defined as follows:

```
H_acc = 1 - H_geo;
```

The bode plot of the obtained complementary filters is shown in Figure

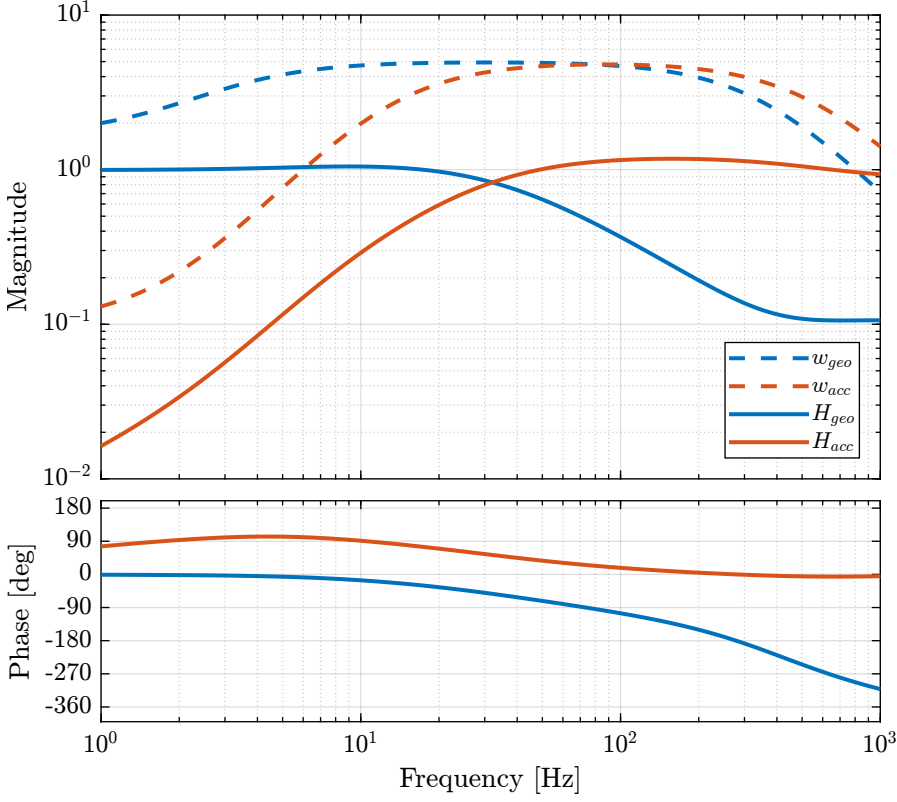


Figure 7.4: Bode plot of the obtained complementary filters using the \mathcal{H}_∞ synthesis

7.5 Obtained Super Sensor Dynamical Uncertainty

The obtained super sensor dynamical uncertainty is shown in Figure 7.5.

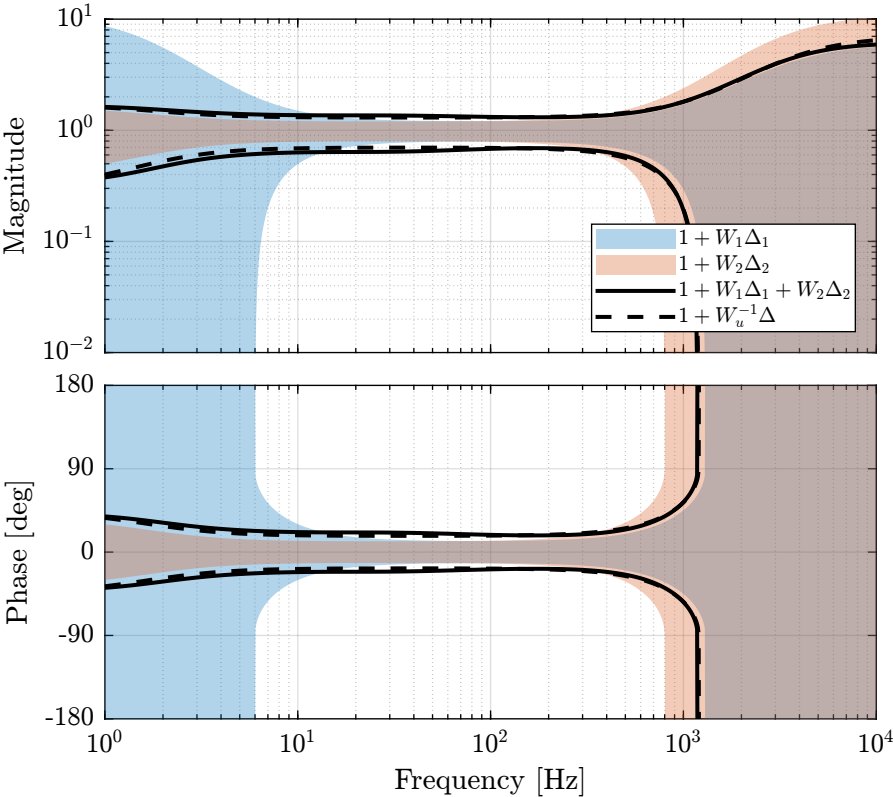


Figure 7.5: Obtained Super sensor dynamics uncertainty

8 Optimal and Robust sensor fusion using the $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis

8.1 Noise and Dynamical uncertainty weights

```
Matlab
N_acc = (s/(2*pi*2000) + 1)^2/(s + 0.1*2*pi)/(s + 1e3*2*pi)/(1 + s/2/pi/1e3); % [m/sqrt(Hz)]
N_geo = 4e-4*((s + 2*pi)/(2*pi*200) + 1)/(s + 1e3*2*pi)/(1 + s/2/pi/1e3); % [m/sqrt(Hz)]
```

```
Matlab
w_acc = createWeight('n', 2, 'G0', 10, 'G1', 0.2, 'Gc', 1, 'w0', 6*2*pi) * ...
        createWeight('n', 2, 'G0', 1, 'G1', 5/0.2, 'Gc', 1/0.2, 'w0', 1300*2*pi);
w_geo = createWeight('n', 2, 'G0', 0.6, 'G1', 0.2, 'Gc', 0.3, 'w0', 3*2*pi) * ...
        createWeight('n', 2, 'G0', 1, 'G1', 10/0.2, 'Gc', 1/0.2, 'w0', 800*2*pi);
```

```
Matlab
wu = inv(createWeight('n', 2, 'G0', 0.7, 'G1', 0.3, 'Gc', 0.4, 'w0', 3*2*pi) * ...
        createWeight('n', 2, 'G0', 1, 'G1', 6/0.3, 'Gc', 1/0.3, 'w0', 1200*2*pi));
```

```
Matlab
P = [wu*w_acc -wu*w_acc;
     0          wu*w_geo;
     N_acc     -N_acc;
     0          N_geo;
     1          0];
```

And the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis is performed.

```
Matlab
[H_geo, ~] = h2hinfsyn(ss(P), 1, 1, 2, [0, 1], 'HINFMAX', 1, 'H2MAX', Inf, 'DKMAX', 100, 'TOL', 1e-3, 'DISPLAY', 'on');
```

```
Matlab
H_acc = 1 - H_geo;
```

8.2 Obtained Super Sensor Noise

```

freqs = logspace(0, 4, 1000);
PSD_Sgeo = abs(squeeze(freqresp(N_geo, freqs, 'Hz'))).^2;
PSD_Sacc = abs(squeeze(freqresp(N_acc, freqs, 'Hz'))).^2;
PSD_Hss = abs(squeeze(freqresp(N_acc*H_acc, freqs, 'Hz'))).^2 + ...
          abs(squeeze(freqresp(N_geo*H_geo, freqs, 'Hz'))).^2;

```

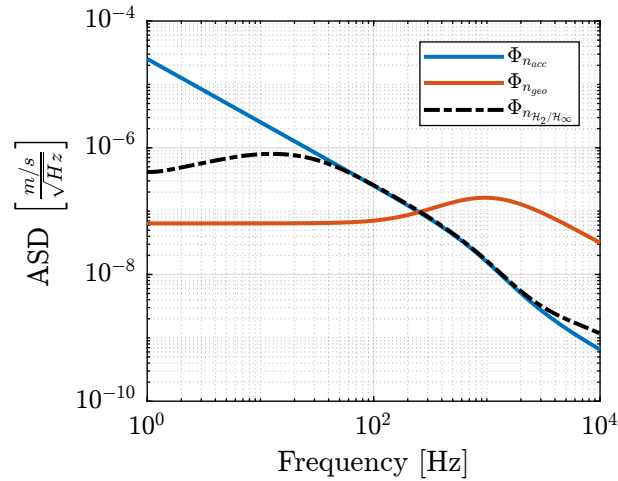


Figure 8.1: Power Spectral Density of the Super Sensor obtained with the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis

8.3 Obtained Super Sensor Dynamical Uncertainty

8.4 Experimental Super Sensor Dynamical Uncertainty

The super sensor dynamics is shown in Figure 8.3.

8.5 Experimental Super Sensor Noise

The obtained super sensor noise is shown in Figure 8.4.

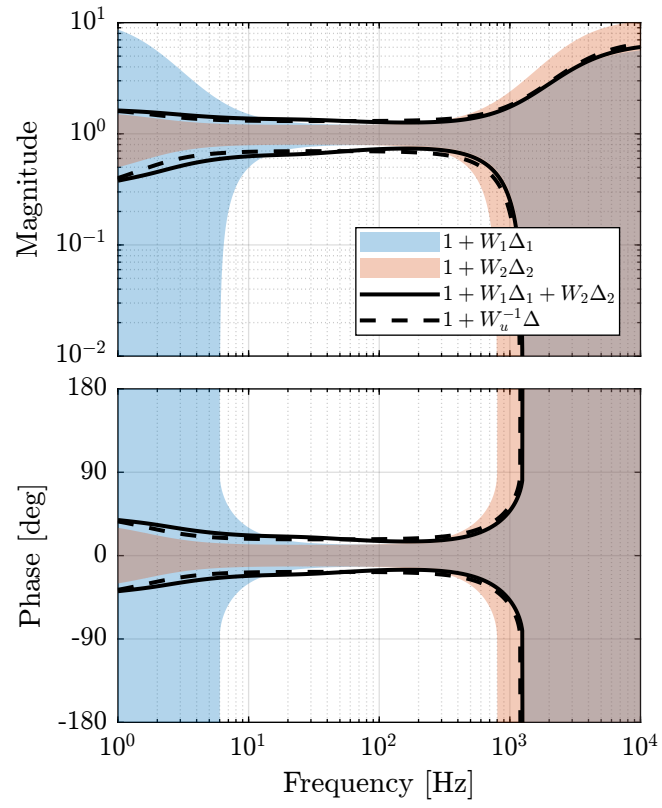


Figure 8.2: Super sensor dynamical uncertainty (solid curve) when using the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

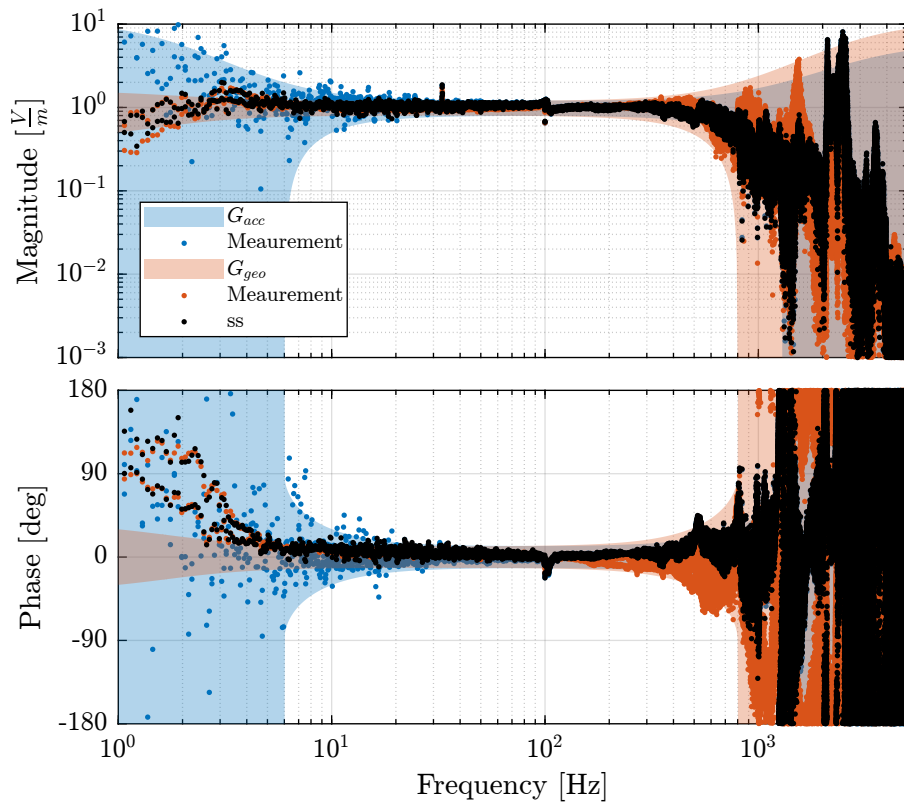


Figure 8.3: Inertial Sensor dynamics as well as the super sensor dynamics

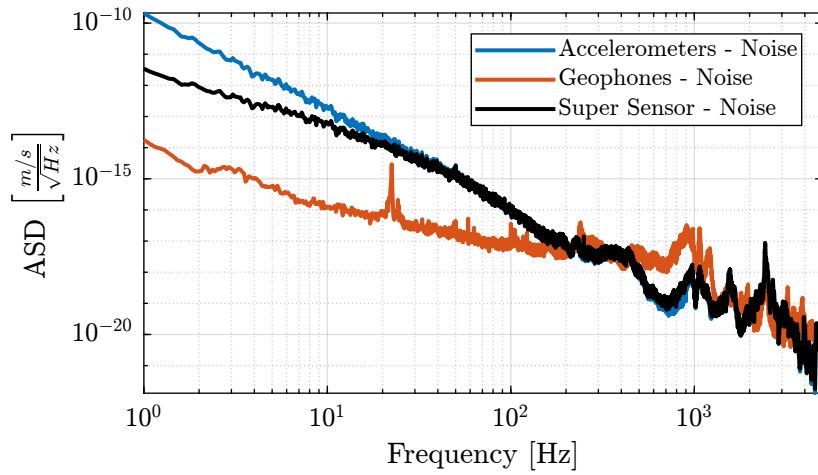


Figure 8.4: ASD of the super sensor obtained using the $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis

9 Matlab Functions

9.1 createWeight

This Matlab function is accessible [here](#).

```
Matlab
function [W] = createWeight(args)
% createWeight -
%
% Syntax: [in_data] = createWeight(in_data)
%
% Inputs:
% - n - Weight Order
% - G0 - Low frequency Gain
% - G1 - High frequency Gain
% - Gc - Gain of W at frequency w0
% - w0 - Frequency at which |W(j w0)| = Gc
%
% Outputs:
% - W - Generated Weight

arguments
    args.n (1,1) double {mustBeInteger, mustBePositive} = 1
    args.G0 (1,1) double {mustBeNumeric, mustBePositive} = 0.1
    args.G1 (1,1) double {mustBeNumeric, mustBePositive} = 10
    args.Gc (1,1) double {mustBeNumeric, mustBePositive} = 1
    args.w0 (1,1) double {mustBeNumeric, mustBePositive} = 1
end

mustBeBetween(args.G0, args.Gc, args.G1);

s = tf('s');

W = (((1/args.w0)*sqrt((1-(args.G0/args.Gc)^(2/args.n))/(1-(args.Gc/args.G1)^(2/args.n))))*s +
↔ (args.G0/args.Gc)^(1/args.n))/((1/args.G1)^(1/args.n)*(1/args.w0)*sqrt((1-(args.G0/args.Gc)^(2/args.n))/(1-(args.Gc/args.G1)^(2/args.n))))*s
↔ + (1/args.Gc)^(1/args.n))^args.n;

end

% Custom validation function
function mustBeBetween(a,b,c)
    if ~(a > b && b > c) || (c > b && b > a)
        eid = 'createWeight:inputError';
        msg = 'Gc should be between G0 and G1.';
        throwAsCaller(MException(eid,msg))
    end
end
end
```

9.2 plotMagUncertainty

This Matlab function is accessible [here](#).

```
Matlab
function [p] = plotMagUncertainty(W, freqs, args)
% plotMagUncertainty -
```

```

%
% Syntax: [p] = plotMagUncertainty(W, freqs, args)
%
% Inputs:
%   - W       - Multiplicative Uncertainty Weight
%   - freqs   - Frequency Vector [Hz]
%   - args    - Optional Arguments:
%       - G
%       - color_i
%       - opacity
%
% Outputs:
%   - p - Plot Handle

arguments
W
freqs double {mustBeNumeric, mustBeNonnegative}
args.G = tf(1)
args.color_i (1,1) double {mustBeInteger, mustBePositive} = 1
args.opacity (1,1) double {mustBeNumeric, mustBeNonnegative} = 0.3
args.DisplayName char = ''
end

% Get defaults colors
colors = get(groot, 'defaultAxesColorOrder');

p = patch([freqs flip(freqs)], ...
          [abs(squeeze(freqresp(args.G, freqs, 'Hz'))).*(1 + abs(squeeze(freqresp(W, freqs, 'Hz')))); ...
            flip(abs(squeeze(freqresp(args.G, freqs, 'Hz'))).*max(1 - abs(squeeze(freqresp(W, freqs, 'Hz'))), 1e-6))], 'w', ...
            'DisplayName', args.DisplayName);

p.FaceColor = colors(args.color_i, :);
p.EdgeColor = 'none';
p.FaceAlpha = args.opacity;

end

```

9.3 plotPhaseUncertainty

This Matlab function is accessible [here](#).

```

function [p] = plotPhaseUncertainty(W, freqs, args)
% plotPhaseUncertainty -
%
% Syntax: [p] = plotPhaseUncertainty(W, freqs, args)
%
% Inputs:
%   - W       - Multiplicative Uncertainty Weight
%   - freqs   - Frequency Vector [Hz]
%   - args    - Optional Arguments:
%       - G
%       - color_i
%       - opacity
%
% Outputs:
%   - p - Plot Handle

arguments
W
freqs double {mustBeNumeric, mustBeNonnegative}
args.G = tf(1)
args.color_i (1,1) double {mustBeInteger, mustBePositive} = 1
args.opacity (1,1) double {mustBeNumeric, mustBePositive} = 0.3
args.DisplayName char = ''
end

% Get defaults colors
colors = get(groot, 'defaultAxesColorOrder');

% Compute Phase Uncertainty

```

```
Dphi = 180/pi*asin(abs(squeeze(freqresp(W, freqs, 'Hz'))));
Dphi(abs(squeeze(freqresp(W, freqs, 'Hz'))) > 1) = 360;

% Compute Plant Phase
G_ang = 180/pi*angle(squeeze(freqresp(args.G, freqs, 'Hz')));

p = patch([freqs flip(freqs)], [G_ang+Dphi; flip(G_ang-Dphi)], 'w', ...
          'DisplayName', args.DisplayName);

p.FaceColor = colors(args.color_i, :);
p.EdgeColor = 'none';
p.FaceAlpha = args.opacity;

end
```

Bibliography

- [1] Aaron Barzilai, Tom VanZandt, and Tom Kenny. “Technique for Measurement of the Noise of a Sensor in the Presence of Large Background Signals”. In: *Review of Scientific Instruments* 69.7 (1998), pp. 2767–2772. DOI: [10.1063/1.1149013](https://doi.org/10.1063/1.1149013). URL: <https://doi.org/10.1063/1.1149013>.